Mathematics for Computer Science
6.042J/18.062J

WELCOME!

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http://theory.lcs.mit.edu/classes/6.042

"Proof, Proofs & More Proofs"

Quick Summary

1. Fundamental concepts of Mathematics.
2. Discrete structures.
3. Discrete probability theory.

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DUE FRIDAY: **Online Tutor**

Reading Problem 1 (RP1):

- *Course Registration*
- *Diagnostic Survey*

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Course Organization

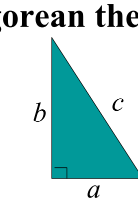
- "Paperless:" All handouts online -- no take-home handouts
- Studio-Lecture Style: mixture of mini-lectures & team problem-solving sessions

Studio Style

Say "hello" to your TA

& the people next to you.

Getting started: Pythagorean theorem

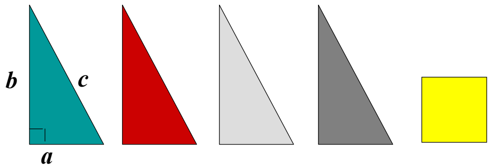


$$a^2 + b^2 = c^2$$

Familiar? Yes!

Obvious? No!

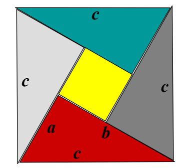
A Cool Proof



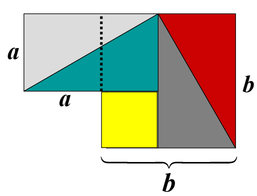
Rearrange into: (i) a $c \times c$ square, and then
(ii) an $a \times a$ & a $b \times b$ square

(Many many proofs: `http://www.cut-the-knot.com`)

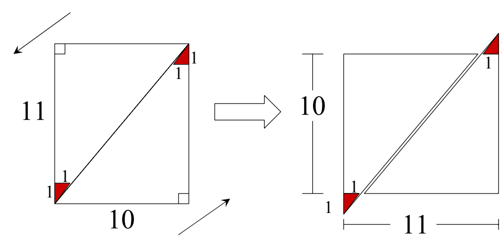
A Cool Proof



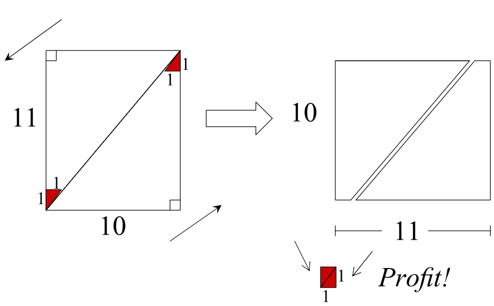
A Cool Proof



A False Proof: Getting Rich By Diagram

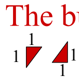


A false proof: getting rich by diagram

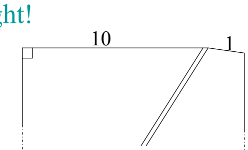


Getting Rich

The bug:

 are not right triangles!

The top and bottom line of the rectangle is not straight!



Another False proof

Theorem: Every quadratic polynomial over $\mathbb{C}$ has two roots.

Proof (by calculation):

The polynomial $ax^2 + bx + c$ has roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Another False proof

Counter-example:

$0x^2 + 0x + 1$ has 0 roots.

$0x^2 + 1x + 1$ has 1 root.

The bug: divide by zero error.
The fix: assume $a \neq 0$.
(Could also say def of "quadratic" requires $a \neq 0$.)

Another false proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The bug: $r_1 = r_2$
The fix: need hypothesis $D \neq 0$ where

$$D ::= \sqrt{b^2 - 4ac}$$

Another false proof

Ambiguity when $D < 0$:
$x^2 + 1$ has roots i, $-i$.
Which is r_1, which is r_2?

Another false proof

The ambiguity causes problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = \left(\sqrt{-1}\right)^2 = -1$$

Moral: "mindless" calculation not safe.

1. Be sure rules are properly applied.
2. Calculation is a risky substitute for understanding.

Consequences of 1= –1

$\frac{1}{2} = -\frac{1}{2}$ (multiply by $\frac{1}{2}$)

$2 = 1$ (add $\frac{3}{2}$)

"Since I and the Pope are clearly 2*, we conclude that I and the Pope are* 1*. That is, I am the Pope."*

-- Bertrand Russell

Consequences of 1= –1

Bertrand Russell (1872 - 1970)

(Picture source: http://www.users.drew.edu/~jlenz/brs.html)

Proof by Contradiction

Theorem: $\sqrt{2}$ *is irrational.*

Proof (by contradiction):

- Suppose $\sqrt{2}$ was rational.
- Choose m, n without common prime factors (always possible) such that

$$\sqrt{2} = \frac{m}{n}$$

- Show that m & n are both even, a contradiction.

Indirect Proof

Theorem: $\sqrt{2}$ *is irrational.*

Proof (by contradiction):

$$\sqrt{2} = \frac{m}{n}$$

$$\sqrt{2}\,n = m$$

$$2n^2 = m^2$$

so m is even.

so can assume $m = 2l$

$$m^2 = 4l^2$$

$$2n^2 = 4l^2$$

$$n^2 = 2l^2$$

so n is even.

Short exercise

Proof assumes that if m^2 is even, then m is even.

Prove it!

Generalizations

Can you prove $\sqrt{3}$ is irrational?

How about $\sqrt[3]{2}$?

CLASS PROBLEMS 1 & 2