Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
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Problem Set 9

Reading: Week 9 Notes, Optional: Rosen §4.3,4.6

Problem 1. A state Senate has fifteen Republicans and ten Democrats. They are creating a committee, which must have six people in it. Since the Republicans have a majority in the Senate, they insist on having a majority on the committee. How many ways are there to form a committee that satisfies the above conditions?

Problem 2. How many positive integers less than $1,000,000$ have exactly one digit equal to 9 and have a sum of digits equal to 13.

Problem 3. Let $B(n)$ denote the number of equivalence relations on n elements.

(a) Show that $B(n) \leq n!$.

(b) Show that $B(n) \geq 2^{n-1}$.

Problem 4. Prove the following identity both by (a) algebraic manipulation and by (b) giving a combinatorial argument:

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

Problem 5. George's 6.042 table has 12 students, who are supposed to break up into 4 groups of 3 students each. George has observed that the students waste too much time trying to form balanced groups, so he decided to pre-assign students to groups and email the group assignments to his students.

How many *different* group assignments are possible? (Clarification: There is no "numbering" of the groups. All that matters is who ends up with whom. Two group assignments are different if they differ in at least one group.)

Problem 6. The *king positioning* in an arrangement of a deck of 52 playing cards is the sequence of numerical positions in the deck, from one to 52, of the four successive kings. For example, the king positioning $(1, 2, 3, 4)$ means all the kings come at the beginning of the deck. The king positioning $(1, 18, 35, 52)$ describes the situation in which the kings are spaced uniformly—with exactly 16 cards between successive kings.

(a) How many king positionings are there?

(b) How many king positionings are there in which *no two kings are adjacent*?

(c) Of the $52!$ possible arrangements of the deck, how many have no two kings adjacent?