

# 6.251/15.081J Recitation 5

## Week 5

### 1 Examples

**Example 1.1.** [1], exercise 4.5.

**Example 1.2.** [1], exercise 4.22.

**Example 1.3.** [1], exercise 4.28.

**Example 1.4.** [1], exercise 4.31.

**Example 1.5.** (Modified from [2], chapter 3). Recall the following LPs considered during the first recitation. Formulate the dual for each part. For **(a)** and **(b)**, show explicitly that the primal unboundedness criterion derived implies dual infeasibility. For **(c)** and **(d)**, show that the optimal value of the dual equals the optimal value of the primal.

**(a)** *Minimizing a linear function over an affine set.*

$$\begin{array}{ll} \text{minimize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b}. \end{array}$$

**(b)** *Minimizing a linear function over a halfspace.*

$$\begin{array}{ll} \text{minimize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & \mathbf{a}^T \mathbf{x} \leq b, \end{array}$$

where  $\mathbf{a} \neq \mathbf{0}$ .

**(c)** *Minimizing a linear function over a rectangle.*

$$\begin{array}{ll} \text{minimize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \end{array}$$

where  $\mathbf{l} \leq \mathbf{u}$ .

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(d) *Minimizing a linear function over the standard simplex.*

$$\begin{aligned} & \text{minimize} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{1}^T \mathbf{x} = 1 \\ & && \mathbf{x} \geq 0. \end{aligned}$$

## References

- [1] Bertsimas, D.; Tsitsiklis, J.N. *Introduction to Linear Optimization*. Athena Scientific, 1997.
- [2] Boyd, S., Vandenberghe, L. *Convex Optimization*. Course reader. Stanford University, 2001.