直角坐标系（x，y，z）中的运动方程

对于τ：

x 轴

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}-\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)+\rho g_x \quad (A)$$

y-轴

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}-\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)+\rho g_y \quad (B)$$

z-轴

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}-\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right)+\rho g_z \quad (C)$$

对于牛顿流体的速度梯度（其中ρ和μ为常数）：

x 轴

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x \quad (D)$$

y-轴

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y \quad (E)$$

z-轴

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z \quad (F)$$

三大基本守恒方程

	I	II	III	Ⅳ		V
守恒方程	局部变化	对流引起的变	扩散引起的变化	产量变化		边界条件
质量	$\frac{\partial c}{\partial t}$	$v\frac{\partial c}{\partial x}$	$D\frac{\partial^2 c}{\partial x^2}$	r	=0	质量转换＝Km=△c
能量	$c_p\rho\frac{\partial T}{\partial t}$	$c_p\rho v\frac{\partial T}{\partial x}$	$\lambda\frac{\partial^2 T}{\partial x^2}$	$\dot{q}$	=0	热转换＝h=△T
动量	$\rho\frac{\partial v}{\partial t}$	$\rho v\frac{\partial v}{\partial x}$	$\eta\frac{\partial^2 v}{\partial x^2}$	f	=0	剪切力＝τa 表面张力=τl

相应的数量（单位体积）	单元	扩散迁移	产量	边界层的传递
质量	c	D	r	$k_m\Delta c$
能量	$c_p\rho T$	λ	$\dot{q}$	$h\Delta T$
动量	ρv	η	f	τ or γL^{-1}

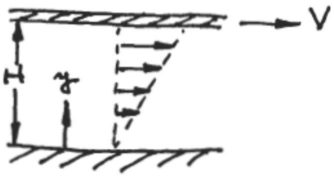
无因次群体系

表3.1中各项的比值	Ⅲ : Ⅰ	Ⅳ : Ⅰ	Ⅴ : Ⅰ	Ⅱ : Ⅲ	Ⅳ : Ⅱ	Ⅴ : Ⅱ	Ⅳ : Ⅲ	Ⅴ : Ⅲ	Ⅳ : Ⅴ
质量	$\frac{Dt}{L^2}$	$\frac{rt}{c}$	$\frac{k_m t}{L}$	$\frac{vL}{D}$ Bo	$\frac{rL}{vc}$ DaⅠ	$\frac{k_m}{v}$ Me	$\frac{rL^2}{Dc}$ DaⅡ	$\frac{k_m L}{D}$ Sh	$\frac{rL}{k_m c}$
能量	$\frac{\lambda t}{c_p\rho L^2}$ Fo	$\frac{\dot{q}t}{c_p\rho T}$	$\frac{ht}{c_p\rho L}$	$\frac{c_p\rho vL}{\lambda}$ Pe	$\frac{\dot{q}L}{c_p\rho Tv}$ DaⅢ	$\frac{h}{c_p\rho v}$ St	$\frac{\dot{q}L^2}{\lambda T}$ DaⅣ	$\frac{hL}{\lambda}$ Nu	$\frac{\dot{q}L}{hT}$
动量	$\frac{\eta t}{\rho L^2}$	$\frac{ft}{\rho v}$	$\frac{\tau t}{\rho vL}$	$\frac{\rho vL}{\eta}$ Re	$\frac{fL}{\rho v^2}$ We	$\frac{\tau}{\rho v^2}$ Fa	$\frac{fL^2}{\eta v}$ Po	$\frac{\tau L}{\eta v}$ Bm	$\frac{fL}{\tau}$

符号代表的意义	
a=单位体积的表面积	γ =表面张力
c=浓度	η =速度
c_p=比热容	λ =热导率
D=扩散系数	ρ =密度
e=电荷	τ =剪切应力
E=弹性模量	ω =角频率
f_{el}=单位体积的电场强度	r=单位体积的反应速率
g=重力加速度	一级反应速率 r=kc
h=传热系数	二级反应速率$r=kc^2$
k=反应速率常数	$\dot{q}$=单位体积的生热率
Km=传质系数	f=单位体积力
l=单位体积的长度	重力 f=gp
L=特征长度	离心力$f=w^2Lp$
P=压力	压力梯度 f=□ P/L
t=时间	弹力 f=E/l
T=温度	表面张力 $f=\frac{\tau}{L^2}$
v=速度	电场力f=e f_{el}
x=长坐标	

数值（看 Gen.Ref.）
Bm = Bingham
Bo = Bodenstein
Da = Damköhler
Fa = Fanning
Fo = Fourier
Me = Merkel
Nu = Nusselt
Pe = Péclet
Po = Poiseuille
Re = Reynolds
Sh = Sherwood
St = Stanton
We = Weber

Couette 流动



$p=\upsilon=\frac{\partial}{\partial x}=0$（简单剪切流）

$$\rho[\frac{\partial u}{\partial t}+\mu\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}]=-\frac{dp}{dx}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\frac{d^2u}{dy^2}=0\rightarrow\frac{du}{dy}=C_1\rightarrow u(y)=C_1y+C_2$$

$$u(0)=0\rightarrow C_2=0,u(u)=V\rightarrow C_1=v/h$$

$$u(y)=\frac{y}{H}V$$

$$\tau_\infty=\frac{F}{A}=\mu(\frac{\partial u}{\partial y})_\omega=\mu\frac{V}{H}$$

生热：

$$Q=\tau\dot{\gamma}=\mu\dot{\gamma}^2=\mu(\frac{V}{H})^2$$

温度分布

$$PC[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}]=Q+k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{d^2T}{dy^2}=-\frac{Q}{k}=-\frac{\mu}{k}(\frac{V}{H})^2\rightarrow\frac{dT}{dy}=-\frac{Q}{k}y+C_1$$

$$T(y)=-\frac{Q}{k}y^2+C_1y+C_2$$

Dirichlet 边界条件：$T(0)=T_1,T(H)=T_2$

Cauchy 边界条件 $T(0)=T_1$

$$H\ \frac{q\,h(T_H-T_a)}{q\,-k\nabla T}\rightarrow\frac{dT(y)}{dy}=-h(T(H)-T_2)$$

牵引流中的温度分布

```
> restart:with(DEtools):
> ode:=diff(T(y),y,y)=-Q/k;
```

$$ode := \frac{\partial^2}{\partial y^2} \mathrm{T}(y) = -\frac{Q}{k}$$

Dirichlet 边界条件

```
> T_f:=simplify(dsolve({ode,T(0)=0,T(1)=0},T(y)));
```

$$T\_f := \mathrm{T}(y) = -\frac{1}{2} \frac{Q\, y\, (y-1)}{k}$$

```
> Digits:=4:k:=1:Q:=1:eq1:=rhs(T_f):
```

Cauchy 边界条件

```
> T_n:=simplify(dsolve({ode,T(0)=0},T(y)));
```

$$T\_n := \mathrm{T}(y) = -\frac{1}{2} y^2 + \_C1\, y$$

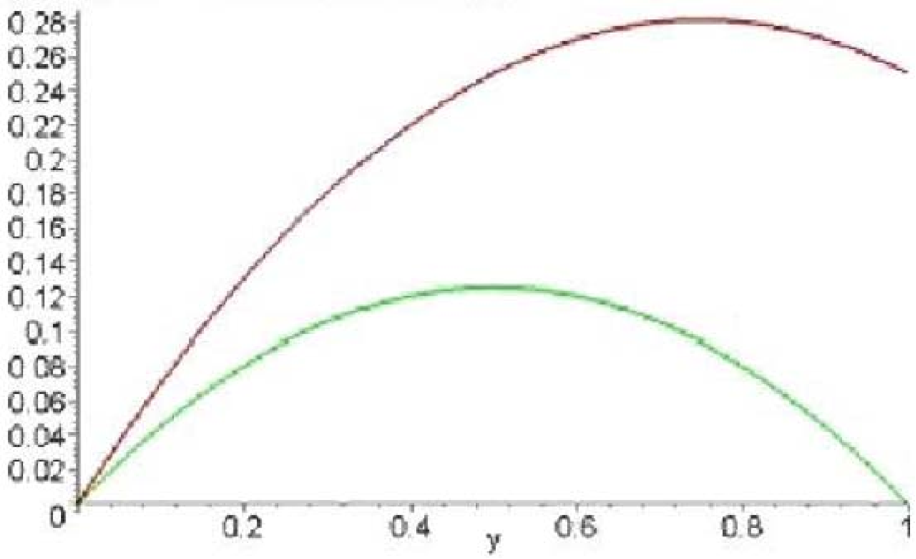
```
> bc_n:=subs(y=1,diff(rhs(T_n),y))=-subs(y=1,rhs(T_n));
```

$$bc\_n := -1 + \_C1 = \frac{1}{2} - \_C1$$

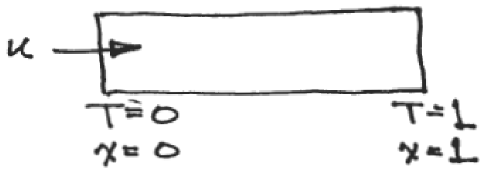
```
> solve(subs(Q=1,bc_n),_C1);
```

$$\frac{3}{4}$$

```
> _C1:=3/4:eq2:=rhs(T_n):
> plot({eq1,eq2},y=0..1,thickness=3);
```



对流



$$PC[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}]=Q+k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$u\frac{dT}{dx}=\frac{k}{pc}\frac{d^2T}{dx^2},\alpha=\frac{k}{pc}$$

$$P_e\frac{dT}{dx}=\frac{d^2T}{dx^2},P_e=\frac{uL}{\alpha}$$

通过扩散和对流进行的热传递

```
> restart:with(DEtools):
> ode:= Pe*diff(T(x),x)=diff(T(x),x,x);
```

$$ode := Pe\left(\frac{\partial}{\partial x}T(x)\right)=\frac{\partial^2}{\partial x^2}T(x)$$

```
> TT:=simplify(dsolve({ode,T(0)=0,T(1)=1},T(x)));
```

$$TT := T(x)=\frac{-1+e^{(Pe\,x)}}{-1+e^{Pe}}$$

```
> eq1:=subs(Pe=1,rhs(TT)):eq5:=subs(Pe=5,rhs(TT)):eq10:=subs(Pe=10,r
  hs(TT)):
> plot({eq1,eq5,eq10},x=0..1,thickness=3);
```