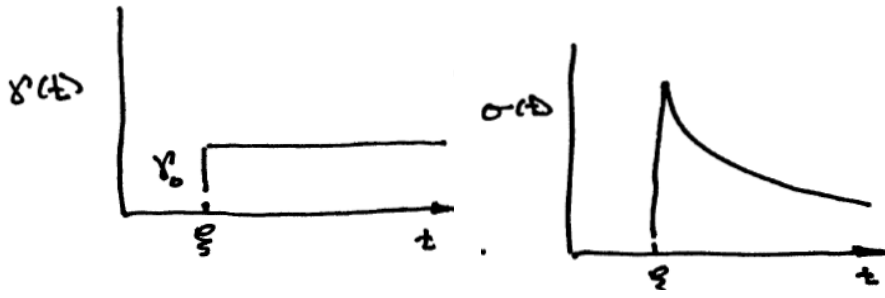
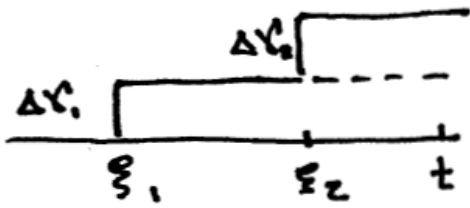


叠加



$$\sigma(t) = G(t - \xi) \cdot \gamma_0$$



$$\sigma(t) = G(t - \xi_1) \cdot \gamma_1 + G(t - \xi_2) \cdot \gamma_2 + \dots$$

$$\sigma(t) = \sum_j G(t - \xi_j) \cdot \Delta \gamma_j \rightarrow \int_{-\infty}^t G(t - \xi) d\gamma$$

$$\sigma(t) = \int_{-\infty}^t G(t - \xi) \frac{d\gamma(\xi)}{d\xi} d\xi$$

Derive complex modulus for Zener solid using superposition

Relaxation modulus

```
> G:=t -> G_R+(G_U - G_R)*exp(-t/tau);
```

$$G := t \rightarrow G_R + (G_U - G_R) e^{\left(-\frac{t}{\tau}\right)}$$

Sinusoidal strain input (unit magnitude)

```
> unprotect(gamma):gamma:=t -> sin(omega*t);
```

$$\gamma := t \rightarrow \sin(\omega t)$$

Superposition integral for stress output

```
> G_star:=int(G(t-xi)*diff(gamma(xi),xi),xi=0..t);
```

Simplify a bit

```
> collect(factor(G_star),sin(omega*t));
```

$$\frac{(-\omega^2 G_U \tau^2 - G_R) \sin(t \omega)}{1 + \omega^2 \tau^2} - \frac{\omega G_U \tau e^{\left(-\frac{t}{\tau}\right)} - \omega G_U \tau \cos(t \omega) + \omega G_R \tau \cos(t \omega) - \omega \tau e^{\left(-\frac{t}{\tau}\right)} G_R}{1 + \omega^2 \tau^2}$$