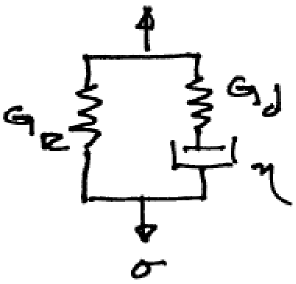
齐纳模型（Zener 模型）



$$\gamma = \gamma_R = \gamma_d$$

$$\sigma = \sigma_R = \sigma_d \text{，} \quad \sigma_R = G_R \gamma$$

$$\dot{\gamma} = \dot{\gamma}_d + \dot{\gamma}_\eta = \frac{1}{G_d}\dot{\sigma} + \frac{1}{\eta}\sigma_d$$

$$G_d \bullet s\bar{\gamma} = s\bar{\sigma}_d + \frac{G_d}{\eta}\bar{\sigma}_d = (s + \frac{1}{\tau_d})\bar{\sigma}_d$$

$$\bar{\sigma}_d = \frac{G_d \bullet s\bar{\gamma}}{s + \frac{1}{\tau_d}}$$

$$\bar{\sigma} = \bar{\sigma}_R + \bar{\sigma}_d = G_R \bar{\gamma} + \frac{G_d \bullet s\bar{\gamma}}{s + \frac{1}{\tau_d}}$$

$$= (G_R + \frac{G_d \bullet s}{s + \frac{1}{\tau_d}})\bar{\gamma} \ (\bar{\sigma} = y\bar{\gamma})$$

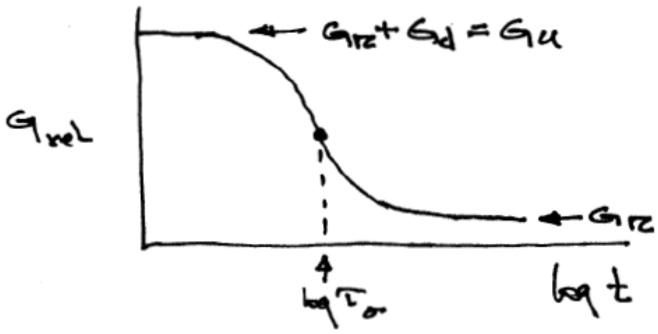
松弛

$$\gamma(t) = \gamma_0 u(t) \rightarrow \bar{\gamma} = \frac{\gamma_0}{s}$$

$$\bar{\sigma} = (G_R + \frac{G_d \bullet s}{s + \frac{1}{\tau_d}})\frac{\gamma_0}{s}$$

$$\frac{\bar{\sigma}_0}{\gamma_0} \equiv \bar{G}_{rel} = \frac{G_R}{s} + \frac{G_d}{s + \frac{1}{\tau_d}}$$

$$G_{rel}(t) = G_R + G_d e^{-\frac{t}{\tau_\sigma}}$$



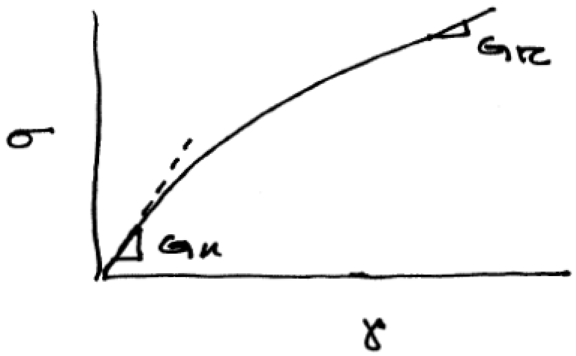
应变速率常数：

$$\gamma(t) = R_r \bullet t \rightarrow \bar{\gamma} = \frac{R_r}{s^2}$$

$$\bar{\sigma} = (G_R + \frac{G_d \bullet s}{s + \frac{1}{\tau_d}}) \bullet \frac{R_r}{s^2} = R_r (\frac{G_R}{s^2} + \frac{G_d}{s(s + \frac{1}{\tau_d})})$$

$$\sigma(t) = G_R \bullet R_r t + G_d R_r \tau_\sigma (1 - e^{-\frac{t}{\tau_\sigma}})$$

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{dt} \bullet \frac{dt}{d\gamma} = G_R + G_d e^{-\frac{t}{\tau_\sigma}} \equiv G_{rel}(t)$$



动态力学分析

Laplace 面剪切

```
> G[L]:=G[R]+ (G[d]*s)/(s+1/tau[sigma]);
```

$$G_L := G_R + \frac{G_d\, s}{s + \frac{1}{\tau_\sigma}}$$

时间面应变：

```
> unprotect(gamma);gamma(t):=gamma[0]*cos(omega*t);
```

$$\gamma(t) := \gamma_0 \cos(\omega t)$$

Laplace 面应变：

```
> with(inttrans):gamma(s):=laplace(gamma(t),t,s);
```

$$\gamma(s) := \frac{\gamma_0\, s}{s^2 + \omega^2}$$

Laplace 面动态模量：

```
> G_bar:=G[L]*gamma(s)/gamma[0];
```

$$G\_bar := \frac{\left(G_R + \frac{G_d\, s}{s + \frac{1}{\tau_\sigma}}\right) s}{s^2 + \omega^2}$$

时间面模量：

```
> G_t:=invlaplace(G_bar,s,t);
```

$$G\_t := \frac{G_d\, e^{\left(-\frac{t}{\tau_\sigma}\right)}}{\omega^2 \tau_\sigma^2 + 1} - \frac{\omega\, \tau_\sigma\, G_d \sin(\omega t)}{\omega^2 \tau_\sigma^2 + 1} + \frac{G_R\, \omega^2 \tau_\sigma^2 \cos(\omega t)}{\omega^2 \tau_\sigma^2 + 1} + \frac{G_R \cos(\omega t)}{\omega^2 \tau_\sigma^2 + 1} + \frac{\omega^2 \tau_\sigma^2\, G_d \cos(\omega t)}{\omega^2 \tau_\sigma^2 + 1}$$

经简化：

```
> 'G(t)'=factor(collect((G_t),cos(omega(t))));
```

$$G(t) = \frac{G_d\, e^{\left(-\frac{t}{\tau_\sigma}\right)} - \omega\, \tau_\sigma\, G_d \sin(\omega t) + G_R\, \omega^2 \tau_\sigma^2 \cos(\omega t) + G_R \cos(\omega t) + \omega^2 \tau_\sigma^2\, G_d \cos(\omega t)}{\omega^2 \tau_\sigma^2 + 1}$$

最后得到：

$$G^{*}=\frac{G_{d}}{1+\omega^{2}\tau_{\sigma}^{2}}e^{\frac{-t}{\tau_{\sigma}}}+\left(G_{R}+\frac{G_{d}\,\omega^{2}\tau_{\sigma}^{2}}{1+\omega^{2}\tau_{\sigma}^{2}}\right)\cos\left(\omega t\right)-\left(\frac{G_{d}\,\omega\tau_{\sigma}}{1+\omega^{2}\ {}_{\sigma}^{2}}\right)$$