$$\rho c\frac{\partial T}{\partial t}=\dot{q}+k\frac{\partial^2 T}{\partial x^2}$$

$$F_0=\frac{k}{\rho c}\bullet\frac{t}{L^2}\qquad Br=\frac{\dot{q}}{k}\bullet\frac{L^2}{T}$$

$$k\sim 0.5\frac{W}{m\bullet k},\rho c\sim 2\times 10^6\frac{J}{m^3\bullet k},\alpha=\frac{k}{\rho c}\sim 10^{-7}\frac{m^2}{s}$$

解法——Laplace 变换

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$$

$$s\bar{T}=\alpha\frac{\partial^2 \bar{T}}{\partial x^2}$$

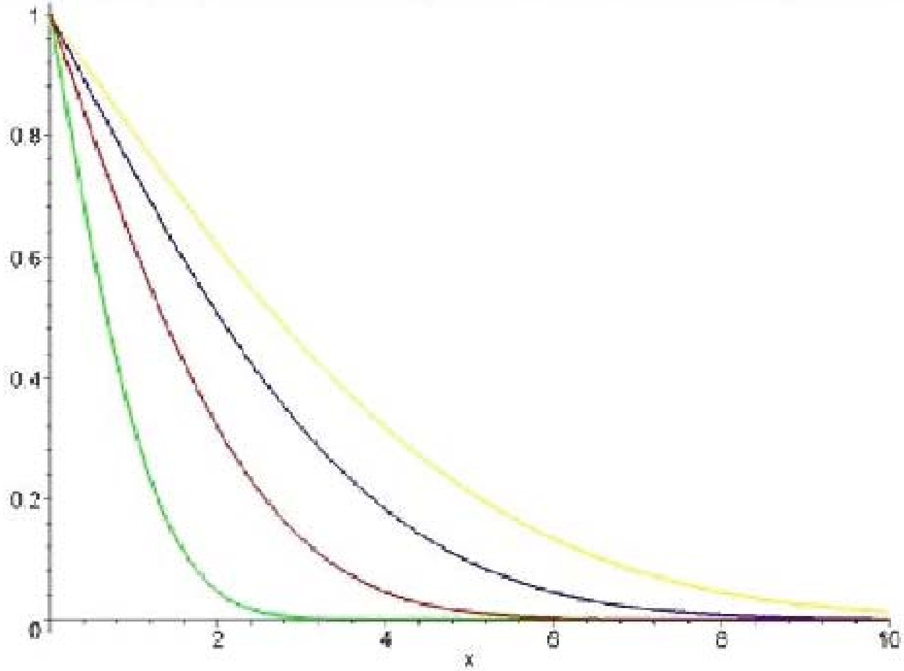
$$T(x,t)=1-erf(\frac{x}{2\sqrt{\alpha t}}),erf(z)=\frac{2}{\Pi}\int_0^z e^{-y^2}dy$$

转变温度曲线

```
> T:=(x,t)-> 1-erf(x/sqrt(2*alpha*t));
```

$$T:=(x,t)\rightarrow 1-\mathrm{erf}\left(\frac{x}{\sqrt{2\alpha t}}\right)$$

```
> alpha:=1:plot({T(x,1),T(x,4),T(x,9),T(x,16)},x=0..10,thickness=3);
```



Maple solution of transient heat equation:

```
> pde:=diff(T(x,t),t)=diff(T(x,t),x,x);
```

$$pde := \frac{\partial}{\partial t}\mathrm{T}(x,t) = \frac{\partial^2}{\partial x^2}\mathrm{T}(x,t)$$

```
> with(DEtools):pdesolve(pde,T(x,t));
```

$$\mathrm{pdesolve}\left(\frac{\partial}{\partial t}\mathrm{T}(x,t) = \frac{\partial^2}{\partial x^2}\mathrm{T}(x,t), \mathrm{T}(x,t)\right)$$

Maple can't get a solution; try simplifying the problem - convert to ODE with Fourier transform::

```
> with(inttrans):ode1:=fourier(pde,x,w);
```

$$ode1 := \frac{\partial}{\partial t}\mathrm{fourier}(\mathrm{T}(x,t),x,w) = -w^2\,\mathrm{fourier}(\mathrm{T}(x,t),x,w)$$

```
> ode:=subs( fourier(T(x,t),x,w)=U(t),ode1);
```

$$ode := \frac{\partial}{\partial t}\mathrm{U}(t) = -w^2\,\mathrm{U}(t)$$

```
> bc:=U(0)=fourier(T_0*Dirac(x),x,w);
```

$$bc := \mathrm{U}(0) = T\_0$$

```
> dsolve({ode,bc},U(t));
```

$$\mathrm{U}(t) = T\_0\,\mathbf{e}^{(-w^2 t)}$$

Invert back to x-plane:

```
> assume(t>0); T(x,t)=simplify(invfourier(rhs(%),w,x));
```

$$\mathrm{T}(x,t\sim) = \frac{1}{2}\,\frac{T\_0\sqrt{\frac{\pi}{t\sim}}\,\mathbf{e}^{\left(-1/4\frac{x^2}{t\sim}\right)}}{\pi}$$

```
> simplify(%);
>
```

$$\mathrm{T}(x,t\sim) = \frac{1}{2}\,\frac{T\_0\,\mathbf{e}^{\left(-1/4\frac{x^2}{t\sim}\right)}}{\sqrt{\pi}\sqrt{t\sim}}$$

有限差分解法

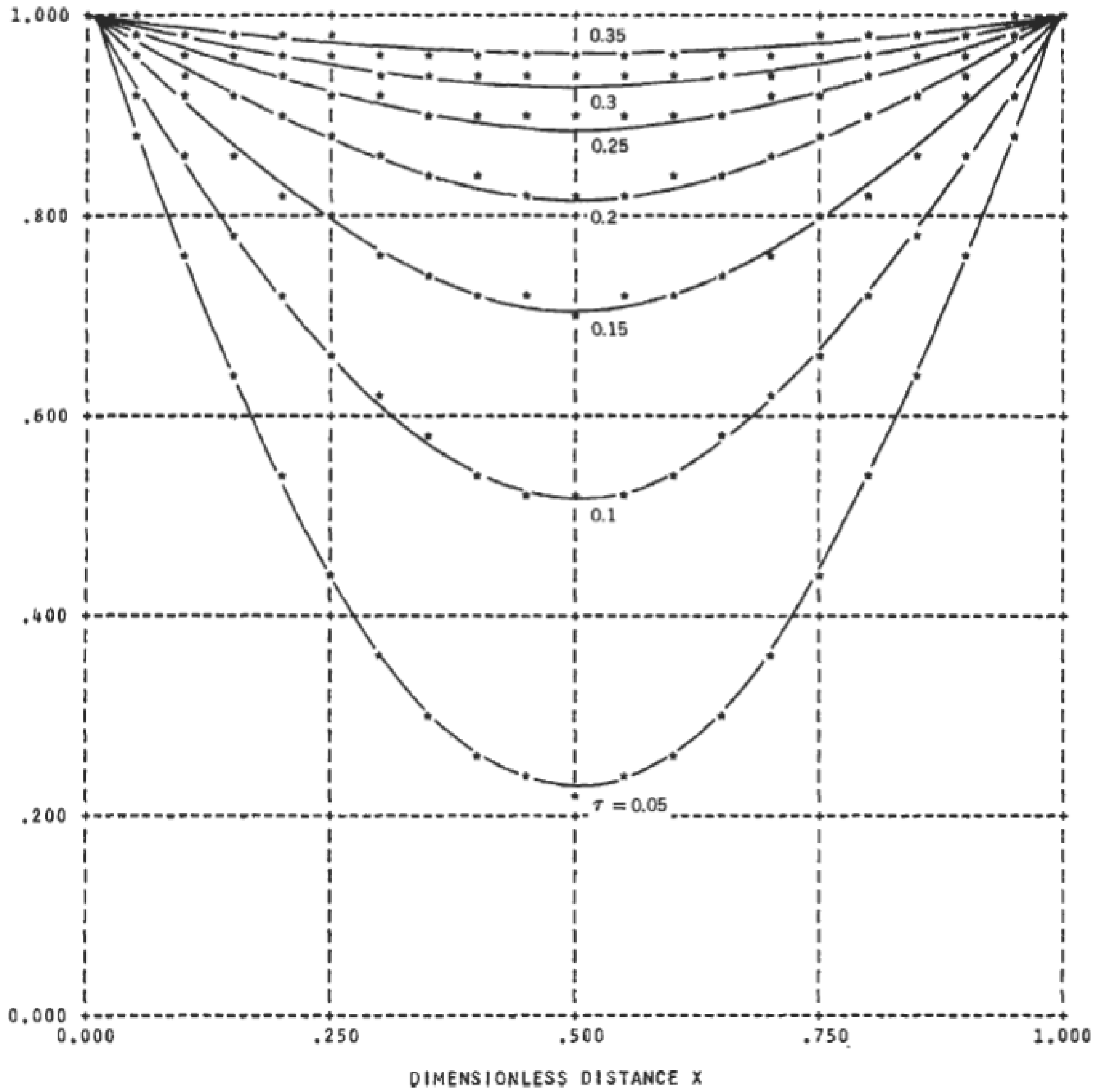
$$\frac{\partial T(x,t)}{\partial t} = \alpha\frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \frac{T_x^{\,t+1} - T_x^{\,t}}{\Delta t} + O(\Delta t)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{x-1}^{\ \ t} - 2T_x^{\,t} + T_{x+1}^{\ \ t}}{(\Delta x)^2} + O[(\Delta x)^2]$$

$$T^{t+1}{}_x = \lambda T_{x-1}^{\ \ t} + (1-2\lambda)T_x^{\,t} + \lambda T_{x+1}^{\ \ t}$$

$$\lambda = \frac{\Delta t}{(\Delta x)^2}$$


1,000
.800
.600
.400
.200
0,000
0.35
0.3
0.25
0.2
0.15
0.1
$\tau = 0.05$
0,000
.250
.500
.750
1,000
DIMENSIONLESS DISTANCE X