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DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING
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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 4

Due in 8 days from its assigned date

Reading

Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials* (John Wiley & Sons, Inc.)
Chapter I, sections 1 – 2.

1. Consider a 500 nm thick aluminum film on a 500 μm thick silicon wafer, 200 mm in diameter. After deposition at 60°C, the wafer is cooled to room temperature. What is the radius of curvature of the film? Is the film in tension or compression? ($E_{\text{Si}} = 150 \text{ GPa}$, $\nu_{\text{Si}} = 0.17$, $\alpha_{\text{Si}} = 3 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$, $E_{\text{Al}} = 69 \text{ GPa}$, $\nu_{\text{Al}} = 0.33$, $\alpha_{\text{Al}} = 23 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$)
2. A bilayer is composed of a 1 mm thick coating ($E = 350 \text{ GPa}$, $\alpha = 9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) on a 5 mm substrate ($E = 70 \text{ GPa}$, $\alpha = 23 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) and has a width of 5 mm. The bilayer has zero curvature at the initial temperature of 20°C. The bilayer is then heated to 100°C. Calculate the resulting curvature and the maximum tensile and compressive stresses. Where do the maximum internal stresses occur?
3. The structure and mechanics of cork: The structure and properties of cork are approximately axisymmetric. The schematics below illustrate a macroscopic view of cork as it exists on a tree, a microscopic look at the arrangement of cork cells, and finally a view of an individual cork cell.

Ignoring the natural variations in cell structure, we can model this cellular material as a linear-elastic solid by

$$\epsilon_{ij} = S_{ijkl} \sigma_{kl}.$$

Axisymmetry reduces the number of independent compliances to five (note that the axis of symmetry is the x_1 axis).

$$\left. \begin{aligned} \epsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1122}\sigma_{33} \\ \epsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33} \\ \epsilon_{33} &= S_{1122}\sigma_{11} + S_{2233}\sigma_{22} + S_{2222}\sigma_{33} \\ \epsilon_{23} &= (S_{2222} - S_{2233})\sigma_{23} \\ \epsilon_{31} &= 2S_{1212}\sigma_{31} \\ \epsilon_{12} &= 2S_{1212}\sigma_{12} \end{aligned} \right\}$$

Measurement of four of the five compliances is straightforward. Simple tensile or compressive tests are done on the cork along the axis of symmetry (x_1 axis) and orthogonal to this direction (x_2 and x_3 axes) to measure the Young's modulus and Poisson's ratio in these directions.

$$\left. \begin{aligned} 1/E_1 &= S_{1111} \\ 1/E_2 &= 1/E_3 = S_{2222} \\ \nu_{12}/E_1 &= \nu_{13}/E_1 = \nu_{21}/E_2 = \nu_{31}/E_2 = -S_{1122} \\ \nu_{23}/E_2 &= \nu_{32}/E_2 = -S_{2233} \end{aligned} \right\}$$

The modulus G_{23} in the x_2, x_3 -plane is obtained from these measurements by

$$1/G_{23} = 2(1 + \nu_{23})/E_2 = 2(S_{2222} - S_{2233}).$$

To determine the fifth compliance value, S_{1212} , which is related to the shear modulus, G_{12} , we rotate the cork through 45° about the x_3 -axis and cut a cube with one face normal to the x_3 -axis, and the other two at 45° to the x_2 -axis and x_1 -axis. A simple compression test in the new x_1 direction then gives a new Young's modulus, E' .

- a. What is the relationship between the shear modulus, G_{12} , and S_{1212} ?
 - b. By rotating the S_{ijkl} through 45° about the x_3 -axis, obtain an expression for S'_{1111} in terms of the compliances in the original orientation.
 - c. Use your answers from parts (a) and (b) to find an expression for the shear modulus G_{12} , in terms of experimental parameters E_1, E_2, ν_{12} , and E' .
 - d. For the isotropic case, what does the equation derived in part (c) become?
4. An isotropic sample of material subjected to a compressive stress σ_z is confined so that it cannot deform in either the x - or y -directions.
- a. Do the stresses occur in the material in the x - and y -directions? If so, how are they related to σ_z ?
 - b. Determine the stiffness $E' = \sigma_z/\epsilon_z$ in the direction of the applied stress in terms of the isotropic elastic constants E and ν for the material. Is E' equal to the elastic modulus from uniaxial loading? Why or why not?
 - c. What happens if the Poisson's ratio for the material approaches 0.5?
5. Consider a flat plate of isotropic material that lies in the x - y plane and which is subjected to applied loading in this plane only. Such a plate is under plane stress, so that $\sigma_z = \tau_{yz} = \tau_{xz} = 0$ MPa.
- a. Does the thickness of the plate usually change when the plate is loaded?
 - b. Under what conditions does the thickness not change? That is, when is this state of plane stress also a state of plane strain?