CHAPTER 17:

MORTGAGE BASICS

(Ch.17, sects.17.1 & 17.2 only)

The "Four Rules" of Loan Payment & Balance Computation. . .

- **Rule 1:** The interest owed in each payment equals the applicable interest rate times the outstanding principal balance (aka: "outstanding loan balance", or "OLB" for short) at the end of the previous period: $\mathbf{INT_t = (OLB_{t-1})r_t}$.

- **Rule 2:** The principal amortized (paid down) in each payment equals the total payment (net of expenses and penalties) minus the interest owed: $\mathbf{AMORT_t = PMT_t - INT_t}$.

- **Rule 3:** The outstanding principal balance after each payment equals the previous outstanding principal balance minus the principal paid down in the payment: $\mathbf{OLB_t = OLB_{t-1} - AMORT_t}$.

- **Rule 4:** The initial outstanding principal balance equals the initial contract principal specified in the loan agreement: $\mathbf{OLB_0 = L}$.

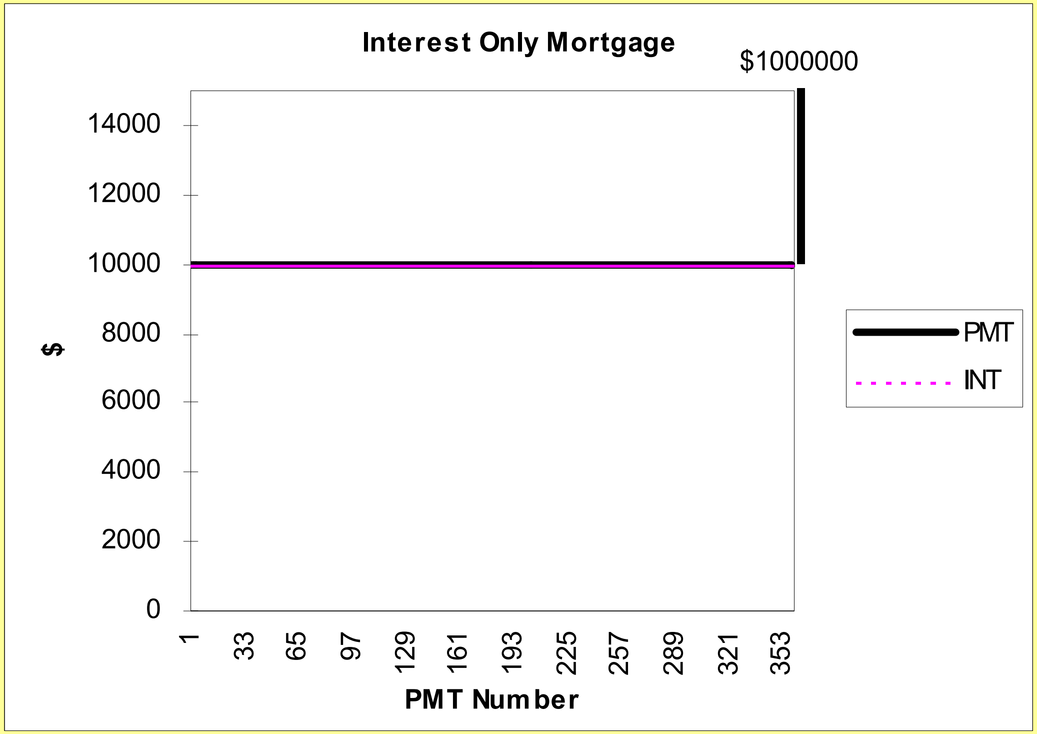
Where:
L = Initial contract principal amount (the "loan amount");
r_t = Contract simple interest rate applicable for payment in Period "t";
INT_t = Interest owed in Period "t";
$AMORT_t$ = Principal paid down in the Period "t" payment;
OLB_t = Outstanding principal balance after the Period "t" payment has been made;
PMT_t = Amount of the loan payment in Period "t".

Know how to apply these rules in a Computer Spreadsheet!

Interest-only loan:

$$PMT_t = INT_t \text{ (or equivalently: } OLB_t = L\text{), for all t.}$$

Exhibit 17-1a: Interest-only Mortgage Payments & Interest Component: $1,000,000, 12%, 30-yr, monthly pmts.



	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$10,000.00	$10,000.00	$0.00	$1,000,000.00
2	$1,000,000.00	$10,000.00	$10,000.00	$0.00	$1,000,000.00
3	$1,000,000.00	$10,000.00	$10,000.00	$0.00	$1,000,000.00
...	...	...	...	...	...
358	$1,000,000.00	$10,000.00	$10,000.00	$0.00	$1,000,000.00
359	$1,000,000.00	$10,000.00	$10,000.00	$0.00	$1,000,000.00
360	$1,000,000.00	$1,010,000.00	$10,000.00	$1,000,000.00	$0.00

How do you construct the pmt & balance schedule in Excel?...

Four columns are necessary:

- **OLB, PMT, INT, AMORT.**
- **(OLB may be repeated at Beg & End of each pmt period to add a 5th col.;)**

• First, "Rule 4" is applied to the 1st row of the OLB column to set initial OLB_0 = L = Initial principal owed;

• Then, the remaining rows and columns are filled in by copy/pasting formulas representing "Rule 1", Rule 2", and "Rule 3",

• Applying one of these rules to each of three of the four necessary columns.

• "Circularity" in the Excel formulas is avoided by placing in the remaining column (the 4th column) a formula which reflects the definition of the type of loan:

- **e.g., For the interest-only loan we could use the $PMT_t = INT_t$ characteristic of the interest-only mortgage to define the PMT column.**
- **Then:**
  - **"Rule 1" is employed in the INT column to derive the interest from the beginning OLB as: $INT_t = OLB_{t-1} * r_t$;**
  - **"Rule 2" in the AMORT column to derive $AMORT_t = PMT_t - INT_t$;**
  - **"Rule 3" in the remainder of the OLB column ($t > 0$) to derive $OLB_t = OLB_{t-1} – AMORT_t$;**
- **(Alternatively, we could have used the $AMORT_t = 0$ loan characteristic to define the AMORT column and then applied "Rule 2" to derive the PMT column instead of the AMORT column.)**

What are some <u>advantages</u> of the <u>interest-only loan</u>?...

- **Low payments.**
- **Payments entirely tax-deductible** *(only marginally valuable for high tax-bracket borrowers)*.
- **If FRM, payments always the same (easy budgeting).**
- **Payments invariant with maturity.**
- **Very simple, easy to understand loan.**

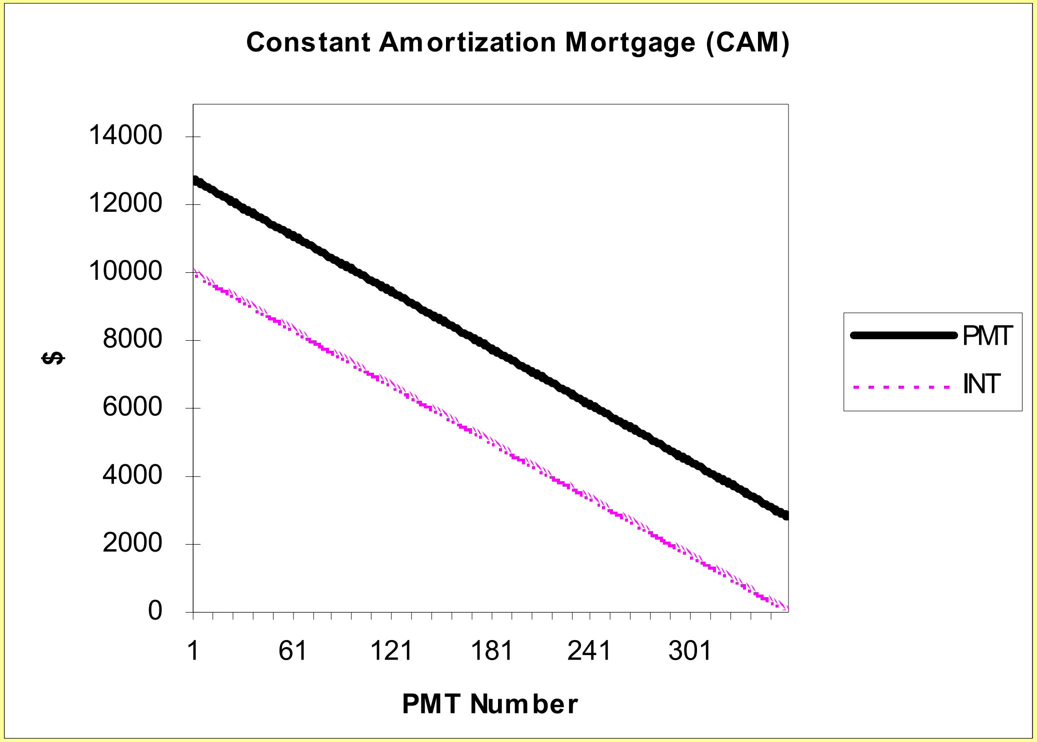
What are some <u>disadvantages</u> of the <u>interest-only loan</u>?...

- **Big "balloon" payment due at end *(maximizes refinancing stress)*.**
- **Maximizes total interest payments *(but this is not really a cost or disadvantage from an NPV or OCC perspective)*.**
- **Has slightly higher *"duration"* than amortizing loan of same maturity *(➔ greater interest rate risk for lender, possibly slightly higher interest rate when yield curve has normal positive slope)*.**
- **Lack of paydown of principle may increase default risk if property value may decline in nominal terms.**

Constant-amortization mortgage (CAM):

$$AMORT_t = L / N, \text{ all } t.$$

Exhibit 17-2: Constant Amortization Mortgage (CAM) Payments & Interest Component: $1,000,000, 12%, 30-yr, monthly pmts.



	Rules 3&4:	Rule 2:	Rule 1:		Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$12,777.78	$10,000.00	$2,777.78	$997,222.22
2	$997,222.22	$12,750.00	$9,972.22	$2,777.78	$994,444.44
3	$994,444.44	$12,722.22	$9,944.44	$2,777.78	$991,666.67
...	...	...	...	...	...
358	$8,333.33	$2,861.11	$83.33	$2,777.78	$5,555.56
359	$5,555.56	$2,833.33	$55.56	$2,777.78	$2,777.78
360	$2,777.78	$2,805.56	$27.78	$2,777.78	$0.00

In Excel, set:

AMORT = 1000000 / 360

Then use "Rules" to derive other columns.

What are some ***advantages*** *of the* ***CAM****?...*

- **No balloon *(no refinancing stress)*.**
- **Declining payments may be appropriate to match a declining asset, or a *deflationary* environment (e.g., 1930s).**
- **Popular for consumer debt (installment loans) on short-lived assets, but not common in real estate.**

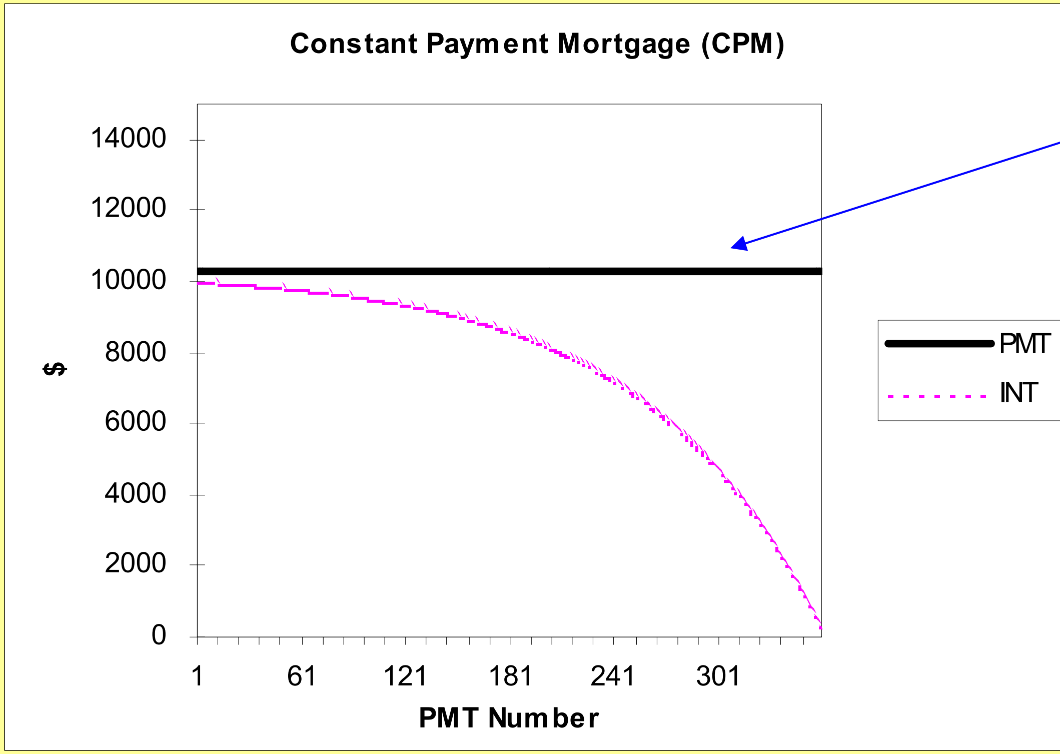
What are some ***disadvantages*** *of the* ***CAM****?...*

- **High initial payments.**
- **Declining payment pattern doesn't usually match property income available to service debt.**
- **Rapidly declining interest component of payments reduces PV of interest tax shield for high tax-bracket investors.**
- **Rapid paydown of principal reduces leverage faster than many borrowers would like.**
- **Constantly changing payment obligations are difficult to administer and budget for.**

The constant-payment mortgage (CPM): *"The Classic"!*

$PMT_t = PMT$, a constant, for all t.

Exhibit 17-3: Constant Payment Mortgage (CPM) Payments & Interest Component: $1,000,000, 12%, 30-year, monthly payments.



Use Annuity Formula to determine constant PMT

Calculator:
360 = N
12 = I/yr
1000000 = PV
0 = FV
Cpt PMT = 10,286

	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$10,286.13	$10,000.00	$286.13	$999,713.87
2	$999,713.87	$10,286.13	$9,997.14	$288.99	$999,424.89
3	$999,424.89	$10,286.13	$9,994.25	$291.88	$999,133.01
...	...	...	...	...	...
358	$30,251.34	$10,286.13	$302.51	$9,983.61	$20,267.73
359	$20,267.73	$10,286.13	$202.68	$10,083.45	$10,184.28
360	$10,184.28	$10,286.13	$101.84	$10,184.28	$0.00

In Excel, set:

=PMT(.01,360,1000000)

Then use "Rules" to derive other columns.

What are some advantages of the CPM?...

- **No balloon *(no refinancing stress)* if fully amortizing.**
- **Low payments possible with long amortization *(e.g., $10286 in 30-yr CPM vs $10000 in interest-only)*.**
- **If FRM, constant flat payments easy to budget and administer.**
- **Large initial interest portion in pmts improves PV of interest tax shields (compared to CAM) for high tax borrowers.**
- **Flexibly allows *trade-off* between pmts, amortization term, maturity, and balloon size.**

What are some disadvantages of the CPM?...

- **Flat payment pattern may not conform to income pattern in some properties or for some borrowers (e.g., in high growth or inflationary situations):**
  - **1st-time homebuyers (especially in high inflation time).**
  - **Turnaround property (needing lease-up phase).**
  - **Income property in general in high inflation time.**

The trade-off in the CPM among:

- **Regular payment level,**
- **Amortization term (how fast the principal is paid down),**
- **Maturity & size of balloon payment…**

Example: Consider 12% $1,000,000 monthly-pmt loan:

What is pmt for 30-yr amortization?

Answer: $10,286.13 (END, 12 P/YR; N=360, I/YR=12, PV=1000000, FV=0, **CPT PMT=**)

What is balloon for 10-yr maturity?

Answer: $934,180 (N=120, **CPT FV=**)

What is pmt for 10-yr amortization (to eliminate balloon)?

Answer: $14,347.09 (FV=0, **CPT PMT=**)

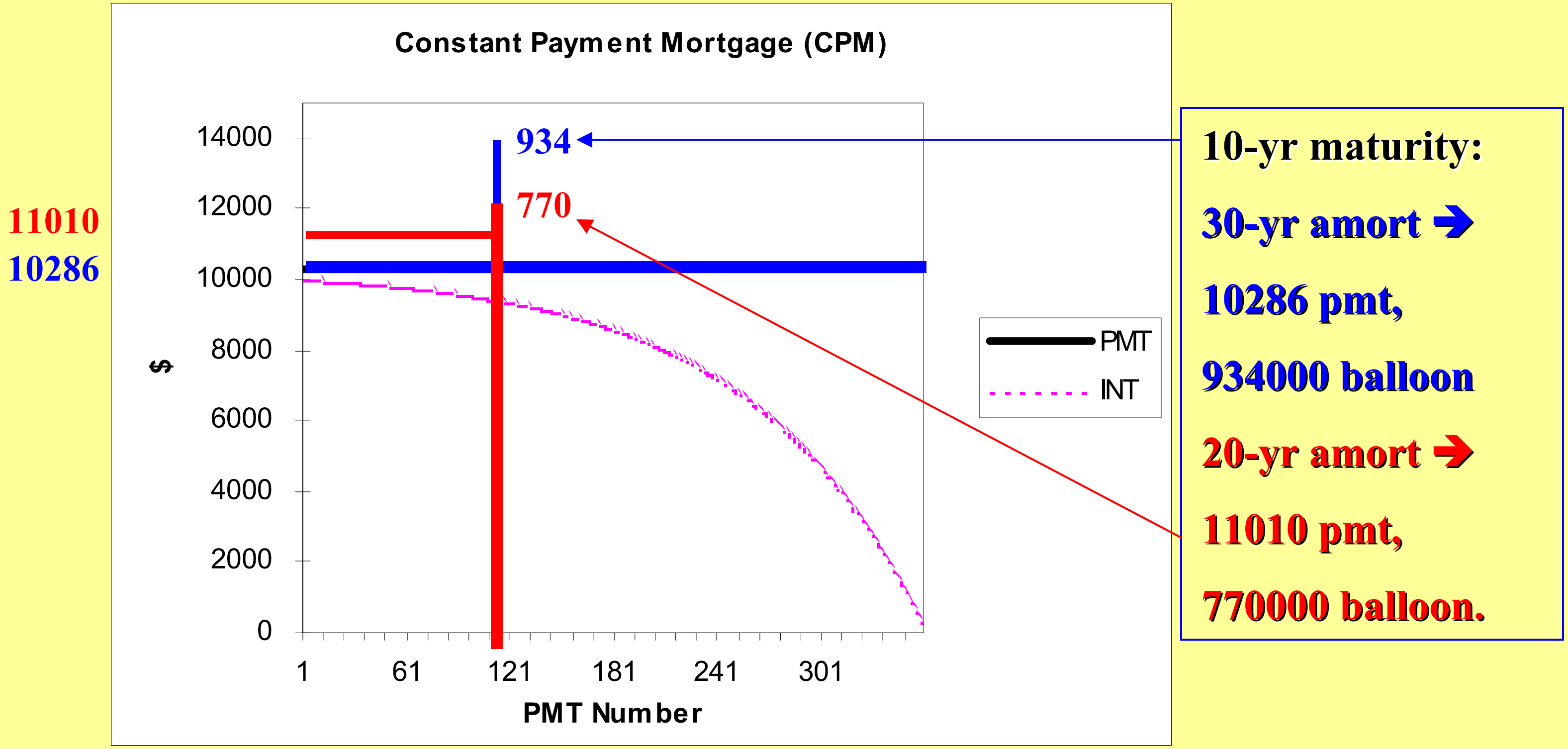
Go back to 30-yr amortization, what is 15-yr maturity balloon (to reduce 10-yr balloon while retaining low pmts)?

Answer: $857,057 (N=360, FV=0, CPT PMT=10286.13, N=180, **CPT FV=**)

<u>The constant-payment mortgage (CPM):</u>

$$PMT_t = PMT, \text{ a constant, for all t.}$$

Exhibit 17-3: Constant Payment Mortgage (CPM) Payments & Interest Component: $1,000,000, 12%, 30-year, monthly payments.



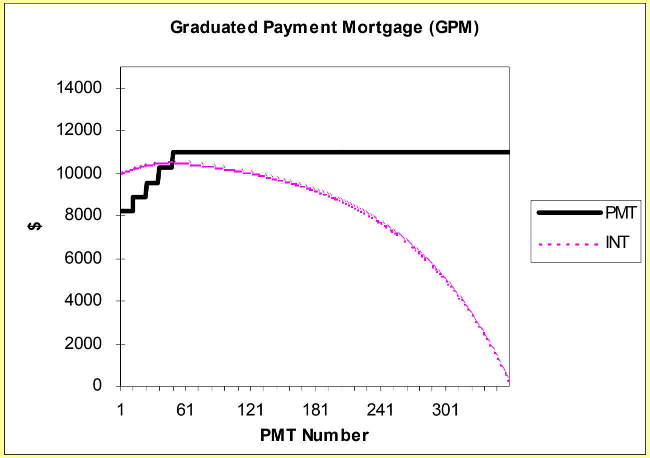
	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$10,286.13	$10,000.00	$286.13	$999,713.87
2	$999,713.87	$10,286.13	$9,997.14	$288.99	$999,424.89
3	$999,424.89	$10,286.13	$9,994.25	$291.88	$999,133.01
...	...	...	...	...	...
358	$30,251.34	$10,286.13	$302.51	$9,983.61	$20,267.73
359	$20,267.73	$10,286.13	$202.68	$10,083.45	$10,184.28
360	$10,184.28	$10,286.13	$101.84	$10,184.28	$0.00

Graduated Payment Mortgage (GPM):

(PMT$_{t+s}$ > PMT$_t$, for some positive value of s and t.)

Allows initial payments to be lower **than they otherwise could be...**

Exhibit 17-4: Graduated Payment Mortgage (GPM) Payments & Interest Component: $1,000,000, 12%, 30-year, monthly payments; 4 Annual 7.5% steps.



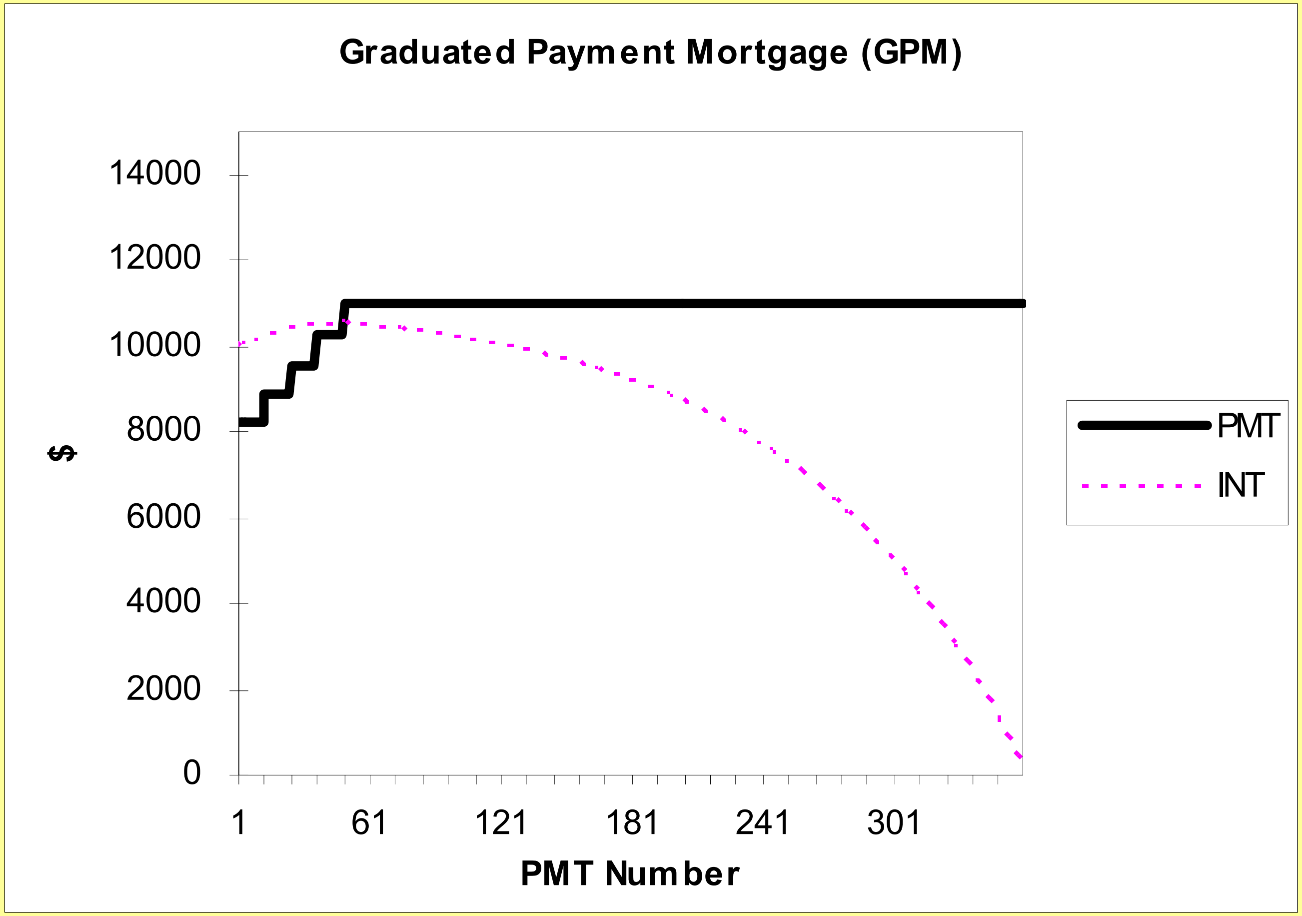
	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$8,255.76	$10,000.00	($1,744.24)	$1,001,744.24
2	$1,001,744.24	$8,255.76	$10,017.44	($1,761.69)	$1,003,505.93
3	$1,003,505.93	$8,255.76	$10,035.06	($1,779.30)	$1,005,285.23
...	...	...	...	...	...
12	$1,020,175.38	$8,255.76	$10,201.75	($1,946.00)	$1,022,121.38
13	$1,022,121.38	$8,874.94	$10,221.21	($1,346.28)	$1,023,467.65
14	$1,023,467.65	$8,874.94	$10,234.68	($1,359.74)	$1,024,827.39
...	...	...	...	...	...
24	$1,037,693.53	$8,874.94	$10,376.94	($1,502.00)	$1,039,195.53
25	$1,039,195.53	$9,540.56	$10,391.96	($851.40)	$1,040,046.92
26	$1,040,046.92	$9,540.56	$10,400.47	($859.91)	$1,040,906.83
...	...	...	...	...	...
36	$1,049,043.49	$9,540.56	$10,490.43	($949.88)	$1,049,993.37
37	$1,049,993.37	$10,256.10	$10,499.93	($243.83)	$1,050,237.20
38	$1,050,237.20	$10,256.10	$10,502.37	($246.27)	$1,050,483.48
...	...	...	...	...	...
48	$1,052,813.75	$10,256.10	$10,528.14	($272.04)	$1,053,085.79
49	$1,053,085.79	$11,025.31	$10,530.86	$494.45	$1,052,591.34
50	$1,052,591.34	$11,025.31	$10,525.91	$499.39	$1,052,091.95
...	...	...	...	...	...
358	$32,425.27	$11,025.31	$324.25	$10,701.05	$21,724.21
359	$21,724.21	$11,025.31	$217.24	$10,808.07	$10,916.15
360	$10,916.15	$11,025.31	$109.16	$10,916.15	$0.00

Graduated Payment Mortgage (GPM):

($PMT_{t+s} > PMT_t$, for some positive value of s and t.)

Allows initial payments to be <u>lower</u> **than they otherwise could be...**

Exhibit 17-4: Graduated Payment Mortgage (GPM) Payments & Interest Component: $1,000,000, 12%, 30-year, monthly payments; 4 Annual 7.5% steps.



Graduated Payment Mortgage (GPM):

($PMT_{t+s} > PMT_t$, for some positive value of s and t.)

Allows initial payments to be ***<u>lower</u>*** **than they otherwise could be...**

Exhibit 17-4: Graduated Payment Mortgage (GPM) Payments & Interest Component:
$1,000,000, 12%, 30-year, monthly payments; **4 Annual 7.5% steps**.

	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):
0					$1,000,000.00
1	$1,000,000.00	$8,255.76	$10,000.00	($1,744.24)	$1,001,744.24
2	$1,001,744.24	$8,255.76	$10,017.44	($1,761.69)	$1,003,505.93
3	$1,003,505.93	$8,255.76	$10,035.06	($1,779.30)	$1,005,285.23
...	...	...	...	...	...
12	$1,020,175.38	$8,255.76	$10,201.75	($1,946.00)	$1,022,121.38
13	$1,022,121.38	$8,874.94	$10,221.21	($1,346.28)	$1,023,467.65
14	$1,023,467.65	$8,874.94	$10,234.68	($1,359.74)	$1,024,827.39
...	...	...	...	...	...
24	$1,037,693.53	$8,874.94	$10,376.94	($1,502.00)	$1,039,195.53
25	$1,039,195.53	$9,540.56	$10,391.96	($851.40)	$1,040,046.92
26	$1,040,046.92	$9,540.56	$10,400.47	($859.91)	$1,040,906.83
...	...	...	...	...	...
36	$1,049,043.49	$9,540.56	$10,490.43	($949.88)	$1,049,993.37
37	$1,049,993.37	$10,256.10	$10,499.93	($243.83)	$1,050,237.20
38	$1,050,237.20	$10,256.10	$10,502.37	($246.27)	$1,050,483.48
...	...	...	...	...	...
48	$1,052,813.75	$10,256.10	$10,528.14	($272.04)	$1,053,085.79
49	$1,053,085.79	$11,025.31	$10,530.86	$494.45	$1,052,591.34
50	$1,052,591.34	$11,025.31	$10,525.91	$499.39	$1,052,091.95
...	...	...	...	...	...
358	$32,425.27	$11,025.31	$324.25	$10,701.05	$21,724.21
359	$21,724.21	$11,025.31	$217.24	$10,808.07	$10,916.15
360	$10,916.15	$11,025.31	$109.16	$10,916.15	$0.00

<u>Graduation characteristics</u> of loan used to derive PMTs based on Annuity Formula.

Then rest of table is derived by applying the "Four Rules" as before.

Once you know what the initial PMT is, everything else follows. . .

Mechanics:
How to calculate the first payment in a GPM...

In principle, we could use the constant-growth annuity formula:

$$PMT_1 = L \Big/ \left(\frac{1-\left(\left(1+g\right)/\left(1+r\right)\right)^N}{r-g} \right)$$

But in practice, only a few (usually annual) "step ups" are made...

For example,

12%, monthly-pmt, 30-yr GPM with 4 annual step-ups of 7.5% each, then constant after year 4:

$$
\begin{aligned}
L = PMT_1(&PV(0.01,12,1) \\
&+ (1.075/1.01^{12})(PV(0.01,12,1) \\
&+ (1.075^2/1.01^{24})(PV(0.01,12,1) \\
&+ (1.075^3/1.01^{36})(PV(0.01,12,1) \\
&+ (1.075^4/1.01^{48})(PV(0.01,312,1))
\end{aligned}
$$

Just invert this formula to solve for "PMT_1".

A potential problem with GPMs:

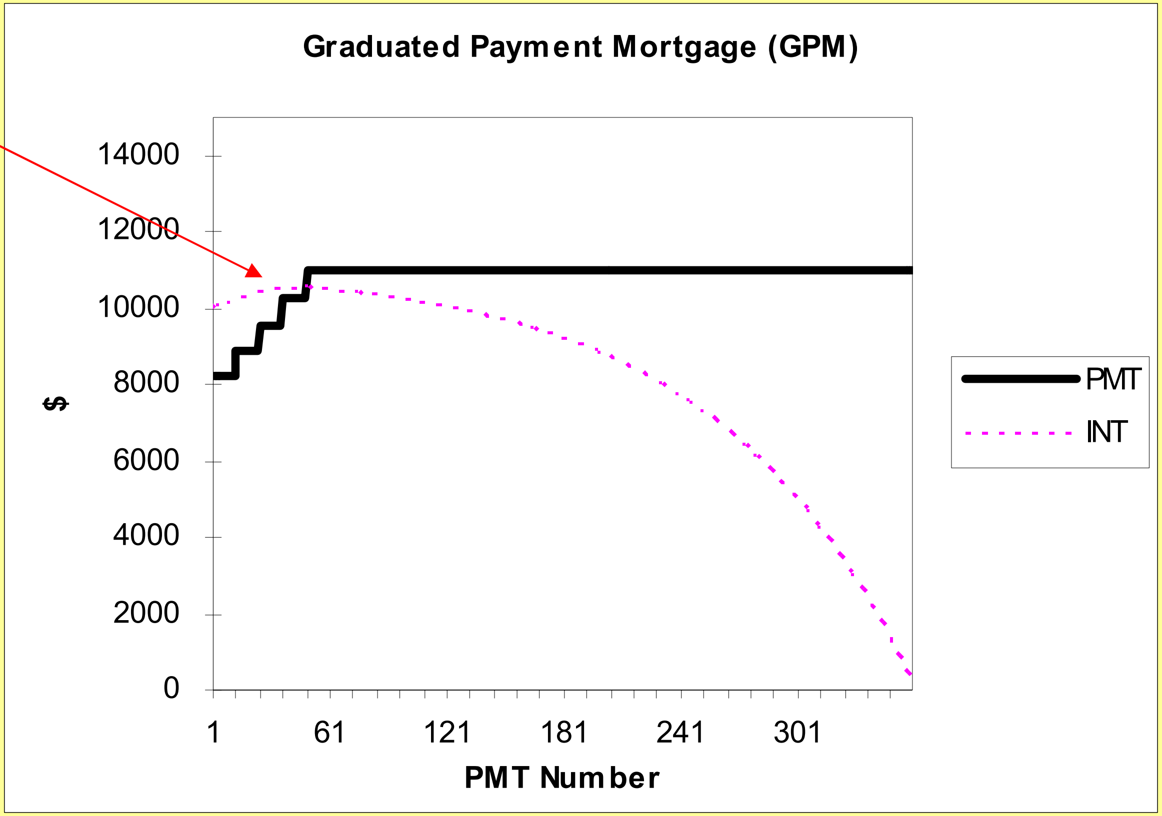
"Negative Amortization". . .

Whenever $PMT_t < INT_t$,

$AMORT_t = PMT_t - INT_t < 0$

e.g., OLB peaks here at $1053086

5.3% above original principal amt.



What are some <u>advantages</u> of the <u>GPM</u>?...

- **Lower initial payments.**
- **Payment pattern that may better match that of income servicing the debt (for turnaround properties, start-up tenants, 1st-time homebuyers, inflationary times).**
- ***(Note: An alternative for inflationary times is the "PLAM" – Price Level Adjusted Mortgage, where OLB is periodically adjusted to reflect inflation, allows loan interest rate to include less "inflation premium", more like a "real interest rate".)***

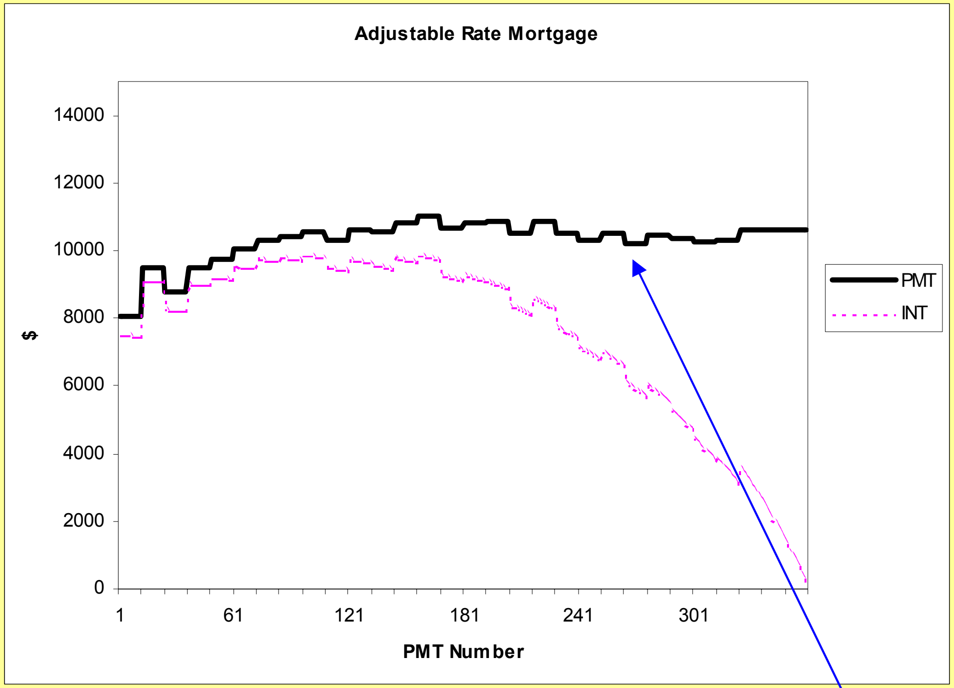
What are some <u>disadvantages</u> of the <u>GPM</u>?...

- **Non-constant payments difficult to budget and administer.**
- **Increased *<u>default risk</u>* due to *<u>negative amortization</u>* and *<u>growing debt service</u>*.**

Adajustable Rate Mortgage (ARM):

r_t may differ from r_{t+s} , for some t & s

Exhibit 17-5: Adjustable Rate Mortgage (ARM) Payments & Interest Component: $1,000,000, 9% Initial Interest, 30-year, monthly payments; 1-year Adjustment interval, possible hypothetical history.



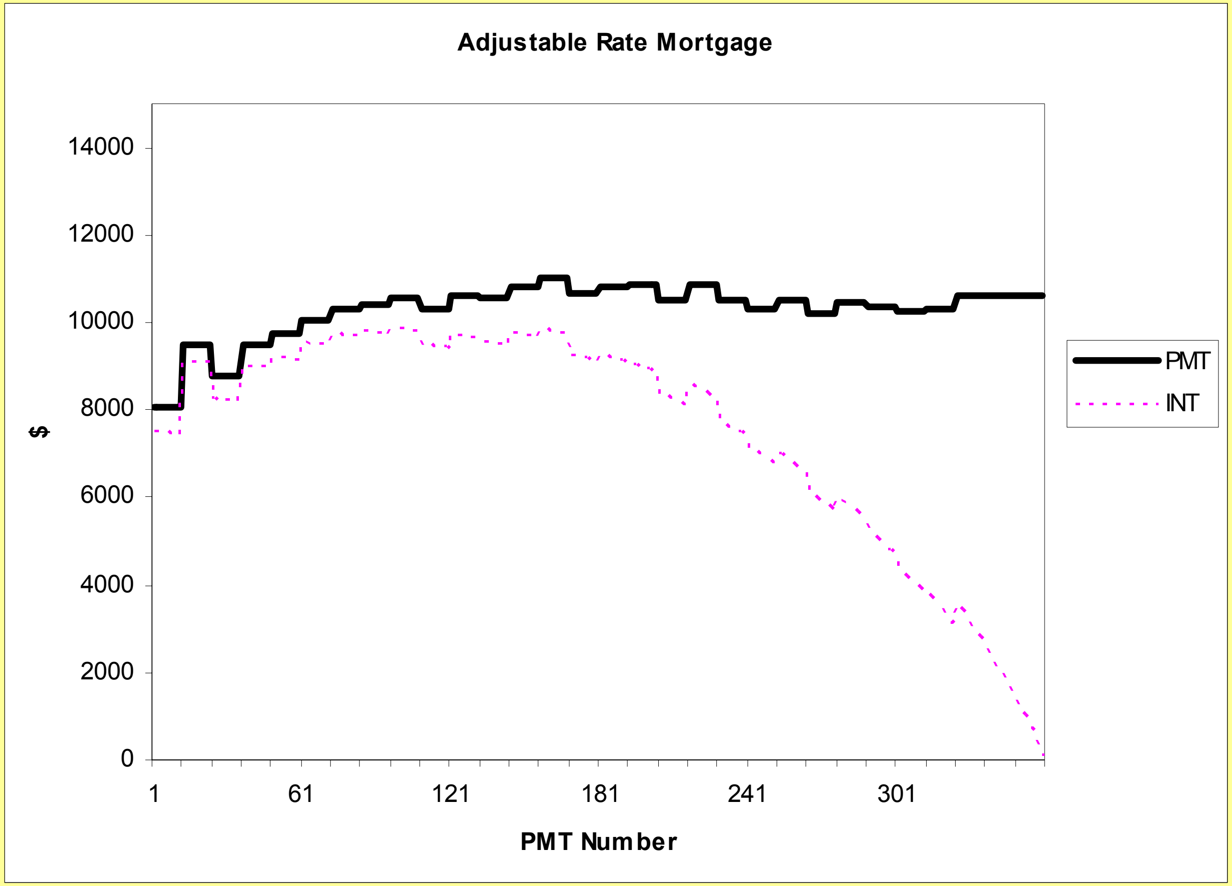
Month#:	Rules 3&4: OLB(Beg):	PMT:	Rule 1: INT:	Rule 2: AMORT:	Rules 3&4: OLB(End):	Applied Rate
0					1000000	
1	1000000	8046.23	7500.00	546.23	999454	0.0900
2	999454	8046.23	7495.90	550.32	998903	0.0900
3	998903	8046.23	7491.78	554.45	998349	0.0900
...	...	...	...	...	...	...
12	993761	8046.23	7453.21	593.02	993168	0.0900
13	993168	9493.49	9095.76	397.73	992770	0.1099
14	992770	9493.49	9092.12	401.37	992369	0.1099
...	...	...	...	...	...	...
24	988587	9493.49	9053.81	439.68	988147	0.1099
25	988147	8788.72	8251.03	537.68	987610	0.1002
26	987610	8788.72	8246.54	542.17	987068	0.1002
...	...	...	...	...	...	...
358	31100	10605.24	356.61	10248.63	20851	0.1376
359	20851	10605.24	239.09	10366.14	10485	0.1376
360	10485	10605.24	120.23	10485.01	0	0.1376

PMT varies over time because market interest rates vary.

Adjustable Rate Mortgage (ARM):

$r_t \neq r_{t+s}$ for some s and t.

Exhibit 17-5: Adjustable Rate Mortgage (ARM) Payments & Interest Component: $1,000,000, 9% Initial Interest, 30-year, monthly payments; 1-year Adjustment interval, possible hypothetical history.



	Rules 3&4:		Rule 1:	Rule 2:	Rules 3&4:	Applied
Month#:	OLB(Beg):	PMT:	INT:	AMORT:	OLB(End):	Rate
0					1000000	
1	1000000	8046.23	7500.00	546.23	999454	0.0900
2	999454	8046.23	7495.90	550.32	998903	0.0900
3	998903	8046.23	7491.78	554.45	998349	0.0900
...	...	...	...	...	...	...
12	993761	8046.23	7453.21	593.02	993168	0.0900
13	993168	9493.49	9095.76	397.73	992770	0.1099
14	992770	9493.49	9092.12	401.37	992369	0.1099
...	...	...	...	...	...	...
24	988587	9493.49	9053.81	439.68	988147	0.1099
25	988147	8788.72	8251.03	537.68	987610	0.1002
26	987610	8788.72	8246.54	542.17	987068	0.1002
...	...	...	...	...	...	...
358	31100	10605.24	356.61	10248.63	20851	0.1376
359	20851	10605.24	239.09	10366.14	10485	0.1376
360	10485	10605.24	120.23	10485.01	0	0.1376

30-year fully-amortizing ARM with:

- **1-year adjustment interval,**
- **9% initial interest rate,**
- **$1,000,000 initial principal loan amount.**

Example:

PMTs 1-12:

360 = N, 9 = I/yr, 1000000 = PV, 0 = FV, *CPT PMT = -8046.23*.

OLB_{12}:

348 = N, *CPT PV = 993168*.

PMTs 13-24:

Suppose applicable int. rate changes to 10.99%.

(with N = 348, PV = 993168, FV = 0, as already):

10.99 = I/yr, *CPT PMT = 9493.49*.

OLB_{24}: 336 = N, *CPT PV = 988147*.

Calculating ARM payments & balances:

1. **Determine the current applicable contract interest rate for each period or adjustment interval (r_t), based on *current market interest rates*.**

2. **Determine the periodic payment for that period or adjustment interval based on the OLB at the beginning of the period or adjustment interval, the number of periods remaining in the amortization term of the loan as of that time, and the current applicable interest rate (r_t).**

3. **Apply the "Four Rules" of mortgage payment & balance determination as always.**

ARM Features & Terminology. . .

Adjustment Interval

e.g., 1-yr, 3-yr, 5-yr: How frequently the contract interest rate changes

Index

The publicly-observable market yield on which the contract interest rate is based.

Margin

Contract interest rate increment above index: $r_t = indexyld_t + margin$

Caps & Floors (in pmt, in contract rate)

- Lifetime: *Applies throughout life of loan.*
- Interval: *Applies to any one adjustment.*

Initial Interest Rate

- "Teaser": *Initial contract rate less than index+margin* $r_0 < indexyld_0 + margin$

- "Fully-indexed Rate": $r_0 = indexyld_0 + margin$

Prepayment Privilege

Residential ARMs are required to allow prepayment w/out penalty.

Conversion Option

Allows conversion to fixed-rate loan (usu. At "prevailing rate").

Because of *caps*, the applicable ARM interest rate will generally be:

r_t = ***MIN*(Lifetime Cap, Interval Cap, Index+Margin)**

Example of *"teaser rate"*:

Suppose:

- **Index = 8% (e.g., current 1-yr LIBOR)**
- **Margin = 200 bps**
- **Initial interest rate = 9%.**

What is the amount of the "teaser"? ***100 bps = (8%+2%) – 9%.***

What will the applicable interest rate be on the loan during the 2nd year if market interest rates <u>remain the same</u> (1-yr LIBOR still 8%)?...

10% = Index + Margin = 8% + 2%,
A 100 bp jump from initial 9% rate.

What are some __advantages__ of the __ARM__?...

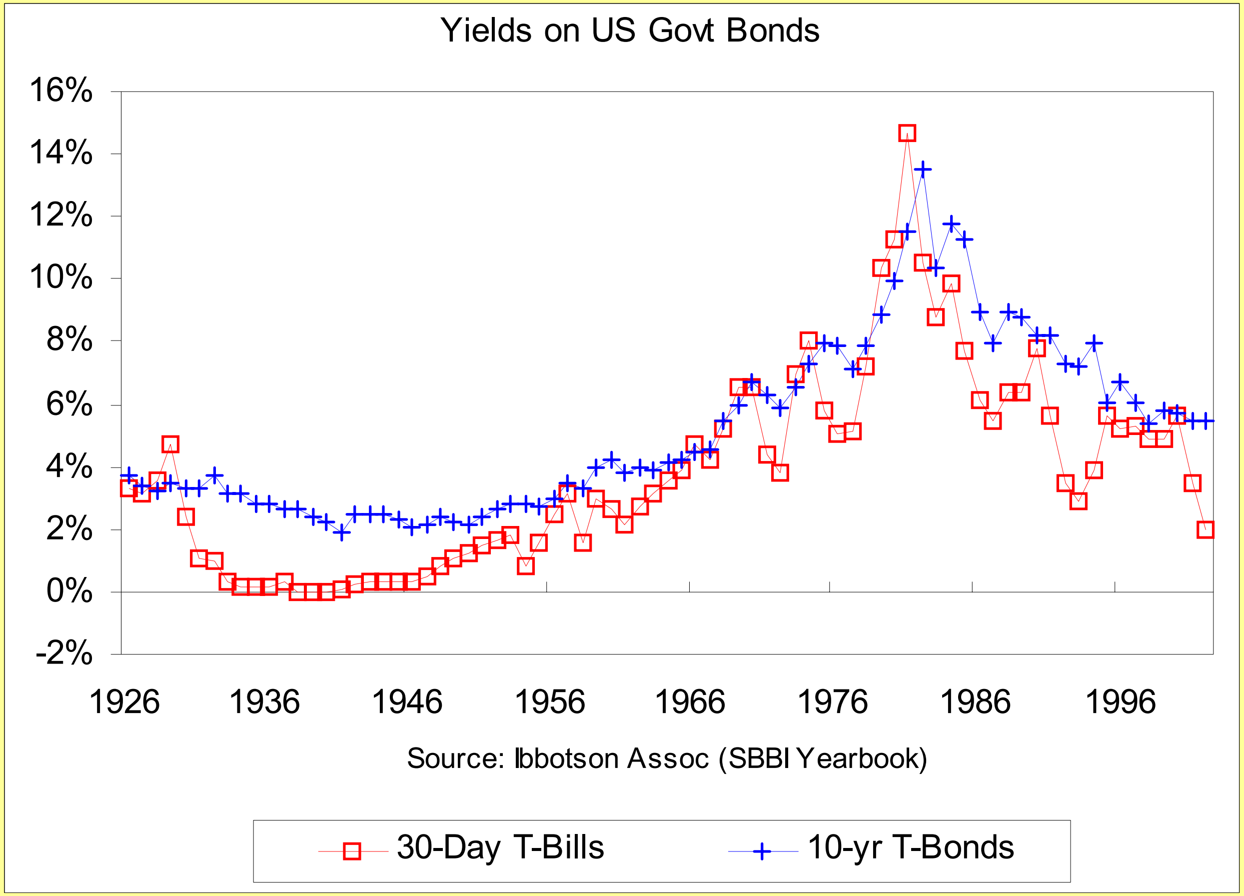
- **Lower *__initial__* interest rate and payments (due to teaser).**
- **Probably *__slightly__* lower __average__ interest rate and payments over the life of the loan, due to typical slight upward slope of bond mkt *__yield curve__* (which reflects *"preferred habitat"* & *"interest rate risk"*).**
- **Reduced interest rate risk for lender (reduces effective "duration", allows pricing off the short end of the yield curve).**
- **Some *hedging* for borrower?... Interest rates tend to rise during "good times", fall during "bad times" (even inflation can be relatively "good" for real estate), so bad news about your interest rate is likely to be somewhat offset by good news about your property or income.**

What are some __disadvantages__ of the __ARM__?...

- **Non-constant payments difficult to budget and administer.**
- **Increased *__interest rate risk__* for *__borrower__* (interest rate risk is transferred from lender to borrower).**
- **As a result of the above, possibly slightly greater *__default risk__*?**
- **All of the above are mitigated by use of:**
  - **Adjustment intervals (longer intervals, less problems);**
  - **Interest rate (or payment) *caps*.**

Some economics behind ARMs (See Chapter 19)

Interest rates are variable, not fully predictable, ST rates more variable than LT rates, more volatility in recent decades . . .



ST rates *usually* (but not always) lower than LT rates:

- ***Upward-sloping "Yield Curve" (avg 100-200 bps).***

Average ("typical") yield curve is *"slightly upward sloping"* (100-200 bps) because:

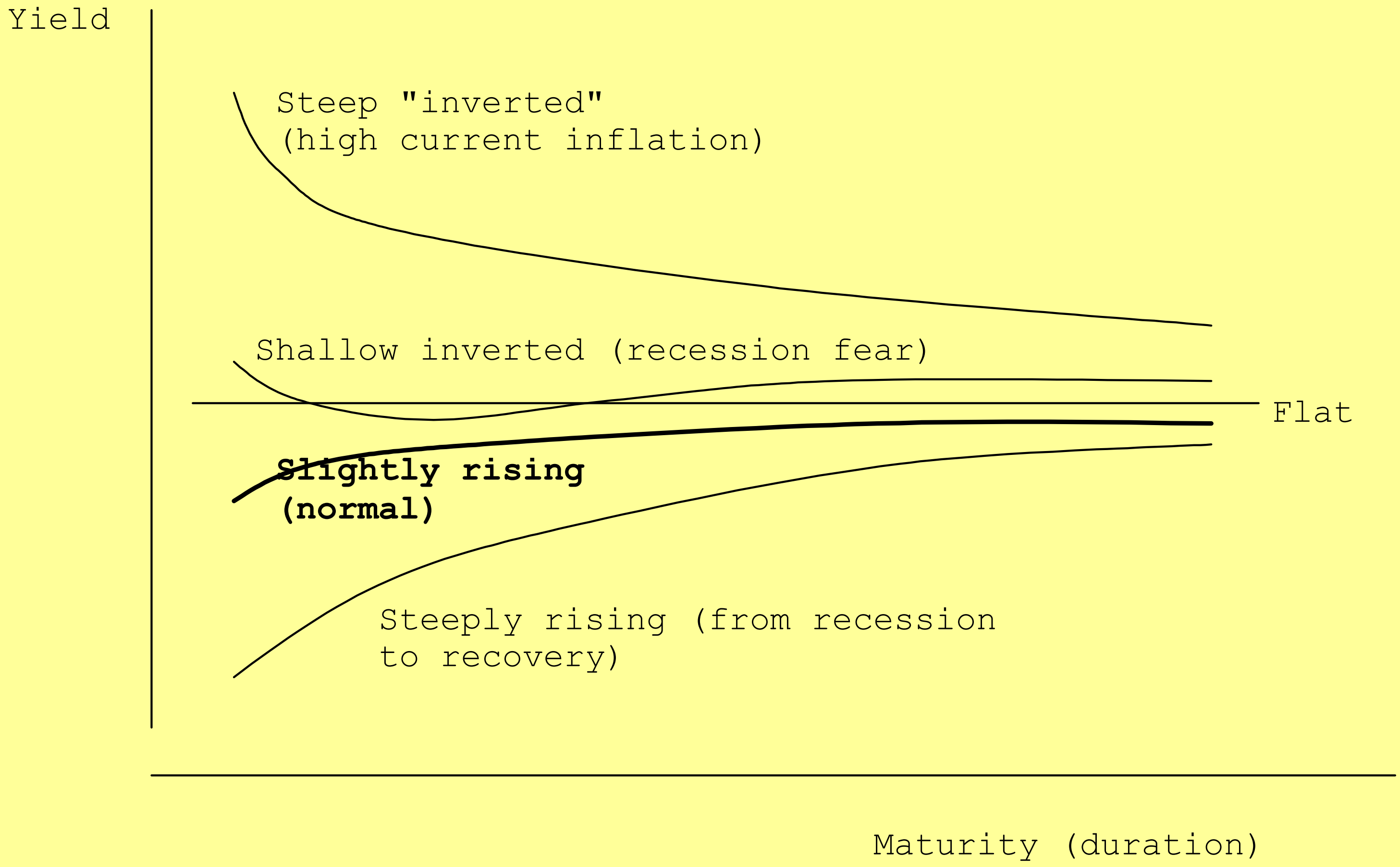
- **Interest Rate Risk:**
  - **Greater volatility in LT bond values and periodic returns (simple HPRs) than in ST bond values and returns:**
  - **➔ LT bonds require greater ex ante risk premium (E[RP]).**
- **"Preferred Habitat":**
  - **More borrowers would rather have LT debt,**
  - **More lenders would rather make ST loans:**
  - **➔ Equilibrium requires higher interest rates for LT debt.**

This is the main fundamental reason why ARMs tend to have slightly lower *lifetime average* interest rates than otherwise similar FRMs, yet not every borrower wants an ARM. Compared to similar FRM:

- ***ARM borrower takes on more interest rate risk,***
- ***ARM lender takes on less interest rate risk.***

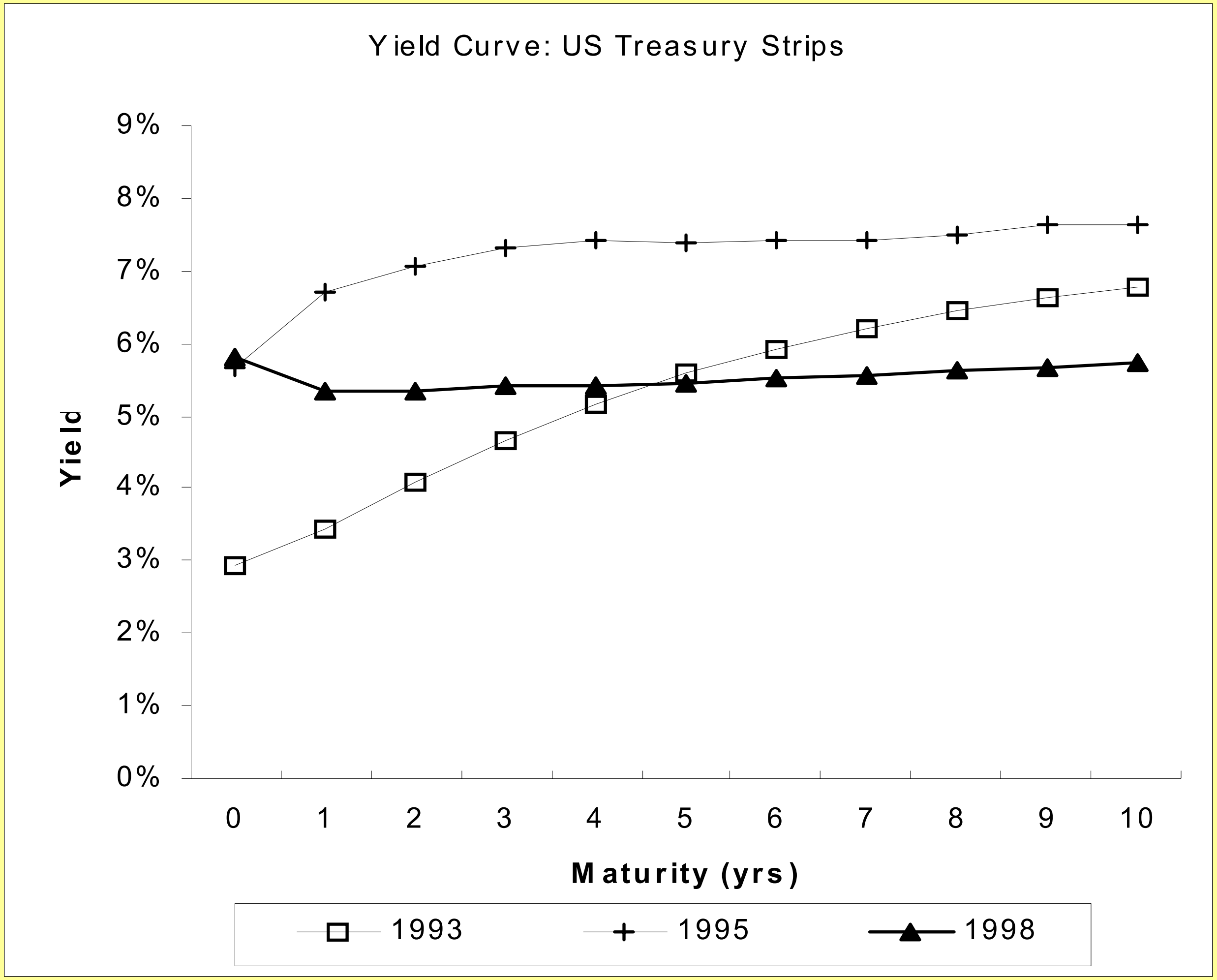
The yield curve is *not always* slightly upward-sloping . . .

Exhibit 19-5:
Typical yield curve shapes . . .



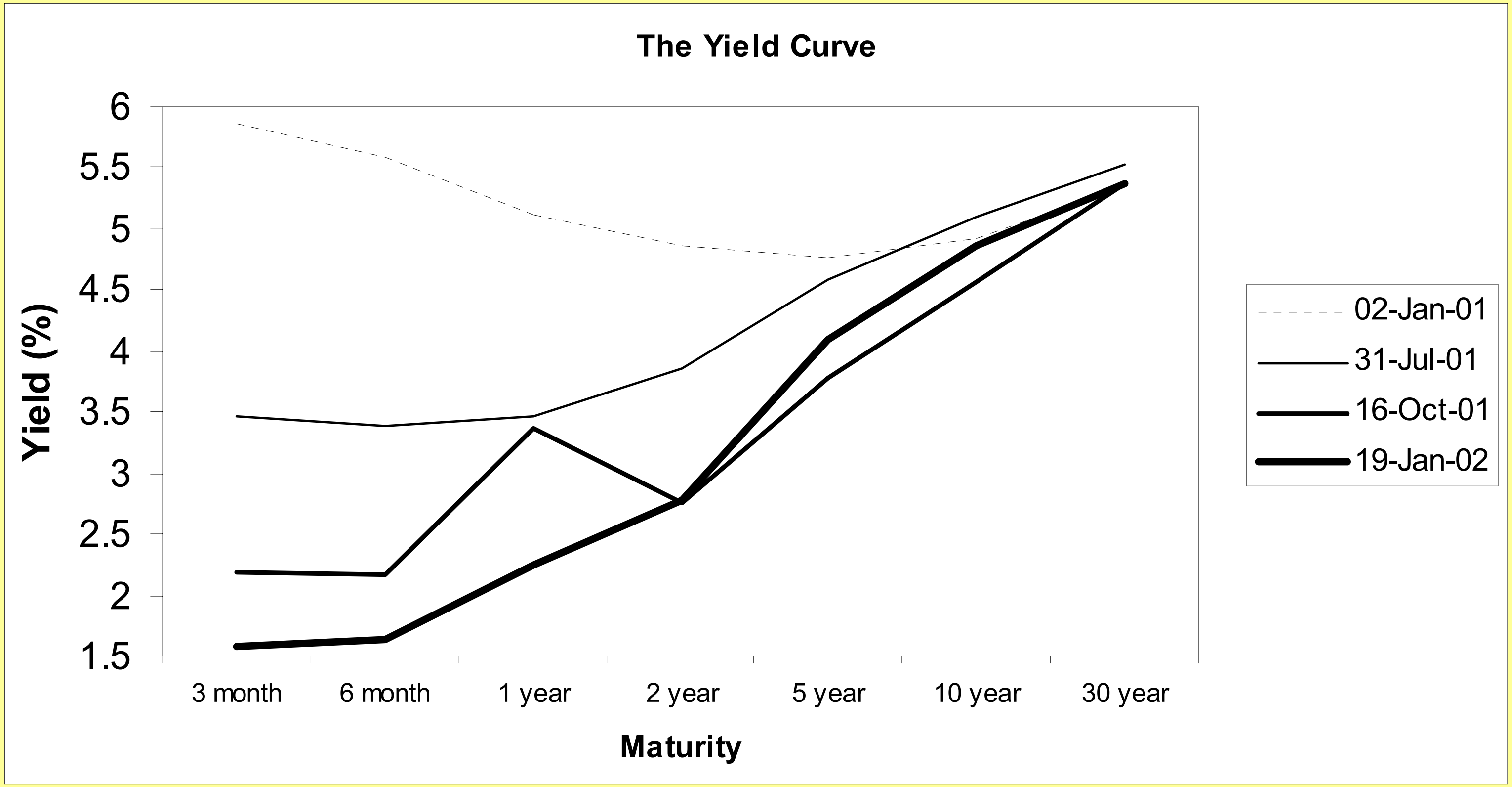
The yield curve is *not always* slightly upward-sloping . . .

The yield curve changes frequently:



The yield curve is *not always* slightly upward-sloping . . .

Here is a more recent example:



Check out *"The Living Yield Curve"* at:

http://www.smartmoney.com/onebond/index.cfm?story=yieldcurve

When the yield curve is *steeply rising* (e.g., 200-400 bps from ST to LT yields), ARM rates may appear *particularly favorable* (for borrowers) relative to FRM rates.

But what do borrowers need to watch out for during such times? . . .

For a long-term borrower, the FRM-ARM differential may be somewhat misleading (ex ante) during such times:

The steeply rising yield curve reflects the *"Expectations Theory"* of the determination of the yield curve:

- **LT yields reflect current *expectations* about *future short-term yields*.**

Thus, ARM borrowers in such circumstances face greater than average risk that their rates will go up in the future.

<u>Design your own custom loan</u> . . .

"Yield" = IRR of the loan.

Most commonly, it is computed as the "Yield to Maturity" (YTM), the IRR over the full contractual life of the loan...

Example:

L = $1,000,000; Fully-amortizing 30-yr monthly-pmt CPM; 8%=interest rate.
(with calculator set for: P/YR=12, END of period CFs...)
360=N, 8%=I/YR, 1000000=PV, 0=FV, Compute: PMT=7337.65.

Solve for "r" :

$$0 = -\$1{,}000{,}000 + \sum_{t=1}^{360} \frac{\$7{,}337.65}{(1+r)^t}$$

Obviously: r = 0.667%, ➔ i = r*m = (0.667%)*12 = 8.00% = YTM.

Here, YTM = "<u>contract interest rate</u>".

This will not always be the case . . .

Suppose loan had 1% (one "point") <u>origination fee</u>
(aka "prepaid interest", "discount points", "disbursement discount")...

Then PV ≠ L:

Borrower only gets (lender only disburses) $990,000.

Solve for "r" in:

$$0 = -\$990{,}000 + \sum_{t=1}^{360} \frac{\$7{,}337.65}{(1+r)^t}$$

Thus: r = 0.6755%, ➔ i = r*m = (0.6755%)*12 = 8.11% = YTM.

360=N, 8%=I/YR, 1000000=PV, 0=FV, Compute: PMT=7337.65;
Then enter* 990000 = PV, *Then CPT I/yr = 8.11%

(Always quote yields to nearest "basis-point" = 1/100th percent.)

Sources of Differences betw YTM vs Contract Interest Rate. . .

1. **“Points” (as above)**
2. **Mortgage Market Valuation Changes over Time...**
   As interest rates change (or default risk in loan changes), the “secondary market” for loans will place different values on the loan, reflecting the need of investors to earn a different “going-in IRR” when they invest in the loan. The market’s **YTM** for the loan is similar to the market’s required **“going-in IRR”** for investing in the loan.

Example:

Suppose interest rates fall, so that the <u>originator</u> of the previous $1,000,000, 8% loan (in the "<u>primary</u> market") can immediately sell the loan in the secondary market for $1,025,000.

i.e., Buyers in the secondary market are willing to pay a "<u>premium</u>" (of $25,000) over the loan's "<u>par</u> value" ("contractual OLB").

Why would they do this? . . .

Mortgage market requires a YTM of 7.74% for this loan:

$$0 = -\$1,025,000 + \sum_{t=1}^{360} \frac{\$7,337.65}{(1+r)^t}$$

r = 0.6452% ➔ i = 0.6452%*12 = 7.74%.

360=N, 1025000=PV, 7337.65=PMT, 0=FV;

Compute: I/YR=7.74%.

This loan has an 8% contract interest rate, but a 7.74% market YTM.
i.e., buyers pay 1025000 because they *<u>must</u>*: "it's the market".

Contract Interest Rates vs YTMs . . .

Contract interest rate *differs* from YTM *whenever*:

- **Current actual CF associated with acquisition of the loan differs from current OLB (or *par value*) of loan.**

At time of loan origination (*primary* market), this results from discounts taken from loan disbursement.

At resale of loan (*secondary* market), YTM reflects market value of loan regardless of par value or contractual interest rate on the loan.

APRs & "Effective Interest Rates". . .

"APR" ("*Annual Percentage Rate*") = YTM from ***lender's*** perspective, at time of loan origination.

("Truth in Lending Act": Residential mortgages & consumer loans.)

Sometimes referred to as "effective interest rate".

CAVEAT (from borrower's perspective):

- **APR is defined from lender's perspective.**
- **Does not include effect of costs of some items required by lender but paid by borrower to 3rd parties (e.g., title insurance, appraisal fee).**
- **These costs may differ across lenders. So lowest effective cost to borrower may not be from lender with lowest official APR.**

Reported APRs for ARMs . . .

The official APR is an *expected yield* (ex ante) at the time of loan origination, based on the *contractual terms* of the loan.

For an ARM, the contract does not pre-determine the future interest rate in the loan. Hence:

The APR of an ARM must be based on a *forecast* of future market interest rates (the *"index"* governing the ARM's applicable rate).

Government regulations require that the "official" APR reported for ARMs be based on a *flat forecast* of market interest rates (i.e., the APR is calculated assuming the index rate remains constant at its current level for the life of the loan).

This is a reasonable assumption when the yield curve has its "normal" slightly upward-sloping shape (i.e., when the shape is due purely to interest rate risk and preferred habitat).

It is a poor assumption for other shapes of the yield curve (i.e., when bond market *expectations* imply that future short-term rates are likely to differ from current short-term rates).

YTMs vs "expected returns". . .

"Expected return"

= Mortgage investor's expected total return (going-in IRR for mortgage investor),
= Borrower's "cost of capital", *E[r]*.

YTM ≠ E[r], for two reasons:

1) YTM based on <u>contractual</u> cash flows, ignoring probability of default. *(Ignore this for now.)*
2) YTM assumes loan remains to <u>maturity</u>, even if loan has prepayment clause...

Suppose previous 30-yr 8%, 1-point (8.11% YTM) loan is expected to be prepaid after 10 years...

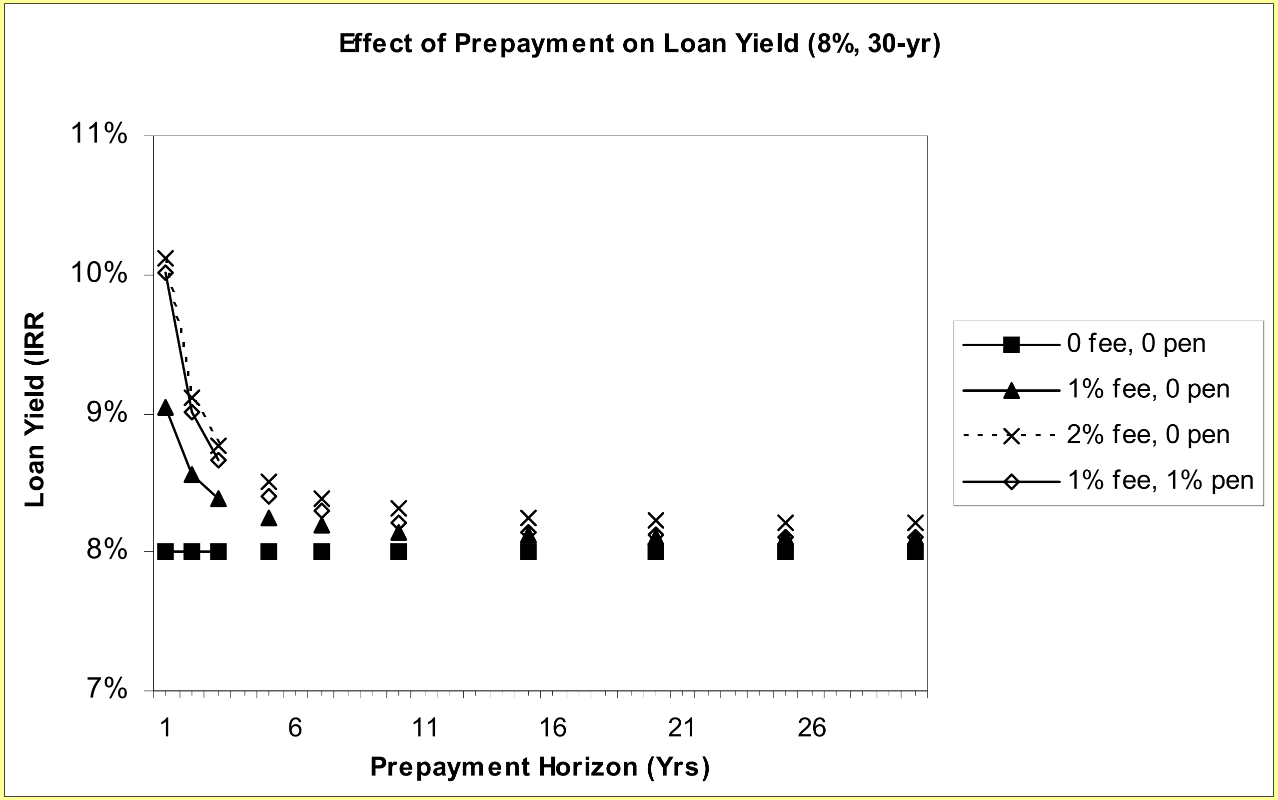
$$0 = -\$990{,}000 + \sum_{t=1}^{120} \frac{\$7{,}337.65}{(1+r)^t} + \frac{\$877{,}247}{(1+r)^{120}}$$

Solve for r = 0.6795%, ➔ E[r]/yr = (0.6795%)*12 = 8.15%.

360=N, 8=I/YR, 1000000=PV, 0=FV;
Compute: PMT= -7337.65.
Then:
120=N; Compute FV= -877247; *then* 990000=PV; Compute:
I/YR=8.15%.

The shorter the prepayment horizon, the greater the effect of any disbursement discount on the realistic yield (expected return) on the mortgage...

Similar (slightly smaller) effect is caused by prepayment penalties.



Prepayment horizon & Expected Return (OCC):

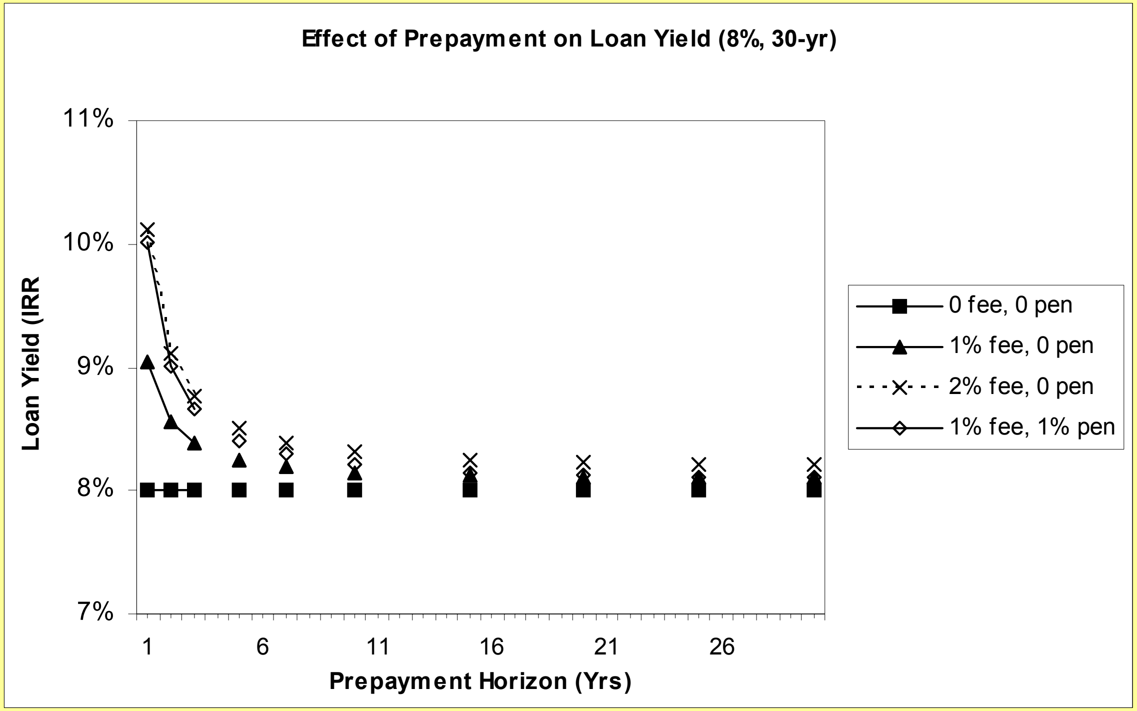


Exhibit 17-2b: Yield (IRR) on 8%, 30-yr CP-FRM:							
	Prepayment Horizon (Yrs)						
Loan Terms:	1	2	3	5	10	20	30
0 fee, 0 pen	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
1% fee, 0 pen	9.05%	8.55%	8.38%	8.25%	8.15%	8.11%	8.11%
2% fee, 0 pen	10.12%	9.11%	8.77%	8.50%	8.31%	8.23%	8.21%
1% fee, 1% pen	10.01%	9.01%	8.67%	8.41%	8.21%	8.13%	8.11%

The **tricky part** in loan yield calculation:

(a) The holding period over which we wish to calculate the yield may not equal the maturity of the loan (e.g., if the loan will be paid off early, so N may not be the original maturity of the loan): $N \neq maturity$;

(b) The actual time-zero present cash flow of the loan may not equal the initial contract principal on the loan (e.g., if there are "points" or other closing costs that cause the cash flow disbursed by the lender and/or the cash flow received by the borrower to not equal the contract principal on the loan, P): $PV = CF_0 \neq L$;

(c) The actual liquidating payment that pays off the loan at the end of the presumed holding period may not exactly equal the outstanding loan balance at that time (e.g., if there is a "prepayment penalty" for paying off the loan early, then the borrower must pay more than the loan balance, so FV is then different from OLB): $CF_N \neq PMT+OLB_N$; *FV to include ppmt penalty*.

So we must make sure that the amounts in the N, PV, and FV registers reflect the actual cash flows…

Example:

Computation of 10-yr yield on 8%, 30-yr, CP-FRM with 1 point discount & 1 point prepayment penalty:

1. **First, enter loan initial contractual terms to compute pmt: 360 ➔ N, 8 ➔ I/yr, 1 ➔ PV, 0 ➔ FV: *CPT* PMT = -.00734.**
2. **Next, change N to reflect actual expected holding period to compute OLB at end: 120 ➔ N, *CPT* FV = -.87725.**
3. **Third step: Add prepayment penalty to OLB to reflect actual cash flow at that time, and enter that amount into FV register: -.87725 X 1.01 = -.88602 ➔ FV.**
4. **Fourth step: Remove discount points from amt in PV register to reflect actual CF_0: RCL PV 1 X .99 = .99 ➔ PV.**
5. **Last: Compute interest (yield) of the actual loan cash flows for the 10-yr hold now reflected in registers: *CPT* I/yr = 8.15%.**

17.2.2 Why do points & fees exist?. . .

1. Compensate brokers who find & filter applications for the lender.
2. Pay back originators for overhead & administrative costs that occur up-front in the "origination" process.

   Above reasons apply to small points and fees.

3. To develop a "mortgage menu", trading off up-front payment vs on-going monthly payment. (Match borrower's payment preferences.)

e.g., All of the following 30-yr loans provide an 8.15% 10-year yield:

Discount Points	Interest Rate	Monthly Payment
0	8.15%	$7444.86
1	8.00%	$7337.65
2	7.85%	$7230.58
3	7.69%	$7124.08

17.2.3 Using Yields to Value Mortgages. . .

The Market Yield is (similar to) the *Expected Return* (going-in) required by *Investors* in the *Mortgage Market*…

Mkt YTM = "OCC" = Discount Rate (applied to contractual CFs)

Thus, Mkt Yields are used to *Value* mortgages (in either the primary or secondary market).

Example:

$1,000,000, 8%, 30-yr-amort, 10-yr-balloon loan again.

How much is this loan worth if the Market Yield is currently 7.5% (= 7.5/12 = 0.625%/mo) MEY (i.e., 7.62% CEY yld in bond mkt)?…

Answer: $1,033,509:

$$\$1{,}033{,}509 = \sum_{t=1}^{120} \frac{\$7{,}337.65}{(1.00625)^t} + \frac{\$877{,}247}{(1.00625)^{120}}$$

(Just the "inverse" of the previous yield computation problem.)

N = 360, I/yr = 8, PV = 1000000, FV = 0, *CPT PMT = -7337.65*; THEN:

N = 120, *CPT FV = -877247*; THEN:

I/yr = 7.5, *CPT PV = 1033509.*

If you know:

1) **Required loan amount (from borrower)**
2) **Required yield (from mortgage market)**

Then you can compute required PMTs, hence, required contract INT & Points . . .

Above example (8%, 30-yr, 10-yr prepayment), suppose mkt yield is 8.5% (instead of 7.5%).

How many POINTs must lender charge on 8% loan (to avoid NPV < 0)?

$$\$967{,}888 = \sum_{t=1}^{120} \frac{\$7{,}337.65}{(1.\underbrace{0070833}_{=8.5\%\,/\,\text{yr}})^{t}} + \frac{\$877{,}247}{(1.0070833)^{120}}$$

Answer: (1000000 – 967888)/1000000 = 3.2% = 3.2 Points.

N = 360, I/yr = 8, PV = 1000000, FV = 0, *CPT PMT = -7337.65*; THEN:

N = 120, *CPT FV = -877247*; THEN:

I/yr = 8.5, *CPT PV = 967888.*

"Bond-Equivalent" & "Mortgage-Equivalent" Rates…

- Traditionally, bonds pay interest *semi-annually* (twice per year).
- Bond interest rates (and yields) are quoted in nominal annual terms (ENAR) assuming semi-annual compounding (m = 2).
- This is often called "bond-equivalent yield" (BEY), or "coupon-equivalent yield" (CEY). Thus:

$$EAR = (1 + BEY/2)^2 - 1$$

"Bond-Equivalent" & "Mortgage-Equivalent" Rates

- Traditionally, mortgages pay interest *monthly*.
- Mortgage interest rates (and yields) are quoted in nominal annual terms (ENAR) assuming monthly compounding (m = 12).
- This is often called "mortgage-equivalent yield" (MEY) Thus:

$$EAR = (1 + MEY/12)^{12} - 1$$

Example:

Yields in the bond market are currently 8% (CEY). What interest rate must you charge on a mortgage (MEY) if you want to sell it at par value in the bond market?

Answer:

7.8698%.

$$EAR = (1 + BEY/2)^2 - 1 = (1.04)^2 - 1 = 0.0816$$

$$MEY = 12\,[(1 + EAR)^{1/12} - 1] = 12\,[(1.0816)^{1/12} - 1] = 0.078698$$

HP-10B	**TI-BAII PLUS**
CLEAR ALL	I Conv
2 P/YR	NOM = 8 ENTER ↓↓
8 I/YR	C/Y = 2 ENTER ↑
EFF% gives 8.16	CPT EFF = 8.16 ↓
12 P/YR	C/Y = 12 ENTER ↑↑
NOM% gives 7.8698	CPT NOM = 7.8698

Example:

You have just issued a mortgage with a 10% contract interest rate (MEY). How high can yields be in the bond market (BEY) such that you can still sell this mortgage at par value in the bond market?

Answer:

10.21%.

$$EAR = (1 + MEY/12)^{12} - 1 = (1.00833)^{12} - 1 = 0.1047$$

$$BEY = 2[(1 + EAR)^{1/2} - 1] = 2[(1.1047)^{1/2} - 1] = 0.1021$$

HP-10B	**TI-BAII PLUS**
CLEAR ALL	I Conv
12 P/YR	NOM = 10 ENTER ↓↓
10 I/YR	C/Y = 12 ENTER ↑
EFF% gives 10.47	CPT EFF = 10.47 ↓
2 P/YR	C/Y = 2 ENTER ↑↑
NOM% gives 10.21	CPT NOM = 10.21